Lecture 6

2021/2022

Microwave Devices and Circuits for Radiocommunications

2021/2022

- 2C/1L, MDCR
- Attendance at minimum 7 sessions (course + laboratory)
- Lectures- associate professor Radu Damian
  - Monday 11-13, P8, ~~Microsoft Teams~~
  - E – 50% final grade, room?
  - problems + (2p atten. lect.) + (3 tests) + (bonus activity)
    - first test L2: 28.02.2022 (t2 and t3 not announced)
    - 3p=+0.5p
  - all materials/equipments authorized

2021/2022

- Laboratory – **associate professor Radu Damian**
  - Thursday 16-18, II.13
  - Friday **8**-12, II.13, odd week
  - L – 25% final grade
    - ADS, 4 sessions
    - Attendance + personal results
  - P – 25% final grade
    - ADS, 3 sessions (-1? 25.02.2022)
    - personal homework

Materials

- RF-OPTO
  - **http://rf-opto.etti.tuiasi.ro**
- **David Pozar, “Microwave Engineering”**, Wiley; 4th edition , 2011
  - 1 exam problem ← Pozar
- Photos
  - sent by ~~email~~/online exam
  - used at lectures/laboratory

Profile photo

- Profile photo – online “exam”

Examene online: 2020/2021

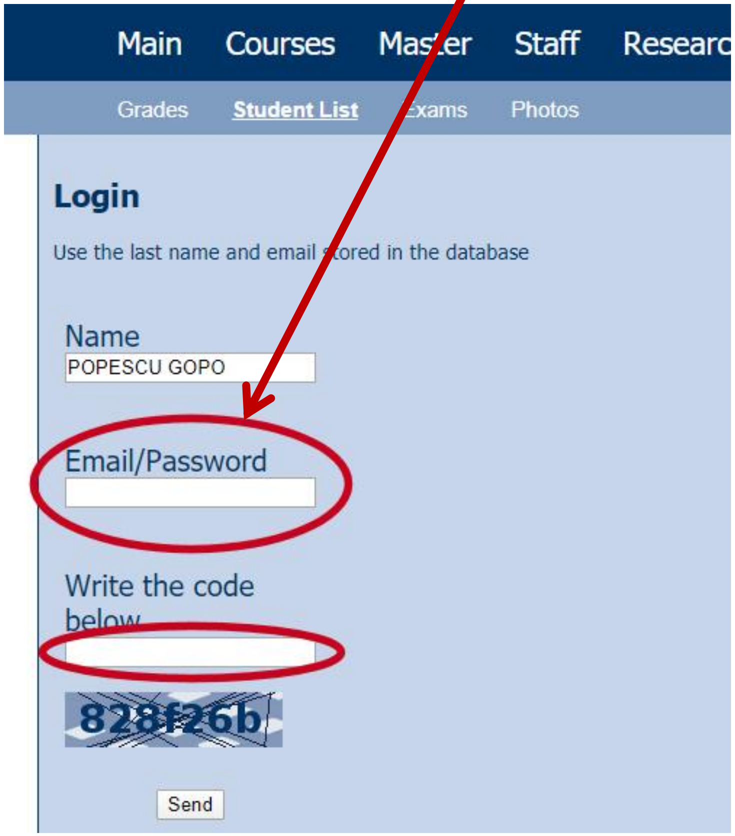
Disciplina: MDC (Microwave Devices and Circuits (Engleza))

Pas 3

Nr.	Titlu	Start	Stop	Text
1	Profile photos	03/03/2021; 10:00	08/04/2021; 08:00	Online "exam" created f ..
2	Mini Test 1 (lecture 2)	03/03/2021; 15:35	03/03/2021; 15:50	The current test consis ..

Online

- access to **online exams** requires the **password** received by email



Online results submission

- many numerical values

Z1	Z2	Z3	Z4	Z5	Z6	Z7
148.33	155.88	202.12	164.35	180.91	30.29	185.19
26.97	153.5	34.64	35.79	55.56	26.212	10.693
0	0	0	0	0	0	0
50	50	50	50	50	50	50

Online results submission

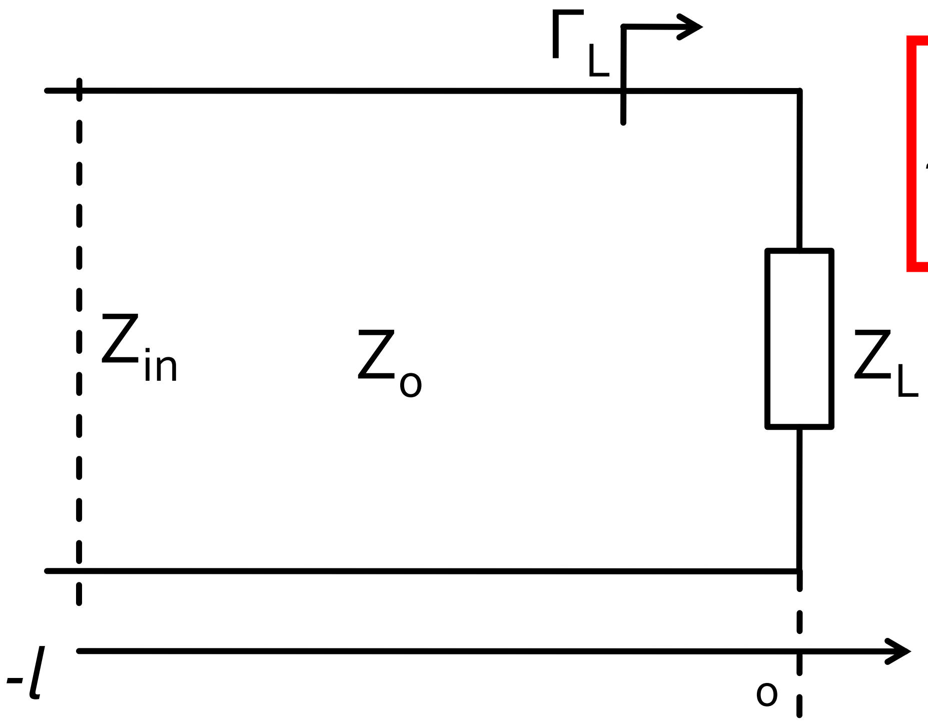
Grade = Quality of the work +

+ Quality of the submission

Important

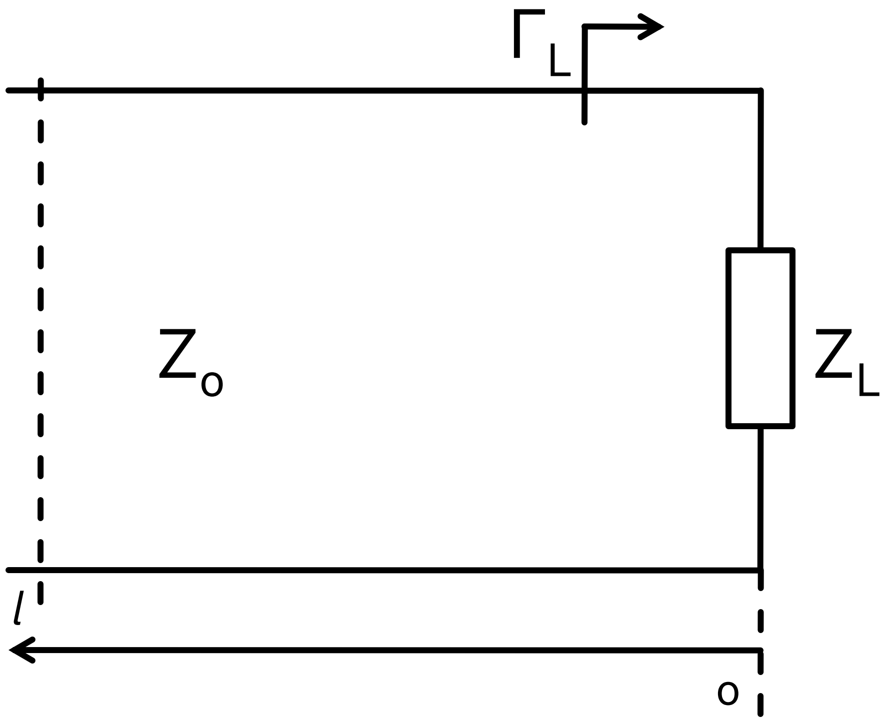
The lossless line

- input impedance of a length ***l*** of transmission line with characteristic impedance $\boldsymbol{Z_o}$, loaded with an arbitrary impedance $\boldsymbol{Z_L}$



$$\boxed{Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}}$$

The lossless line



$$V(z)=V_0^+e^{-j\cdot\beta\cdot z}+V_0^-e^{j\cdot\beta\cdot z}$$

$$I(z)=\frac{V_0^+}{Z_0}e^{-j\cdot\beta\cdot z}-\frac{V_0^-}{Z_0}e^{j\cdot\beta\cdot z}$$

$$Z_L=\frac{V(0)}{I(0)} \qquad Z_L=\frac{V_0^++V_0^-}{V_0^+-V_0^-}\cdot Z_0$$

- voltage reflection coefficient

$$\boxed{\Gamma=\frac{V_0^-}{V_0^+}=\frac{Z_L-Z_0}{Z_L+Z_0}}$$

- Z_o real

The lossless line

$$V(z)=V_0^+\cdot\left(e^{-j\cdot\beta\cdot z}+\Gamma\cdot e^{j\cdot\beta\cdot z}\right) \qquad I(z)=\frac{V_0^+}{Z_0}\cdot\left(e^{-j\cdot\beta\cdot z}-\Gamma\cdot e^{j\cdot\beta\cdot z}\right)$$

- time-average Power flow along the line

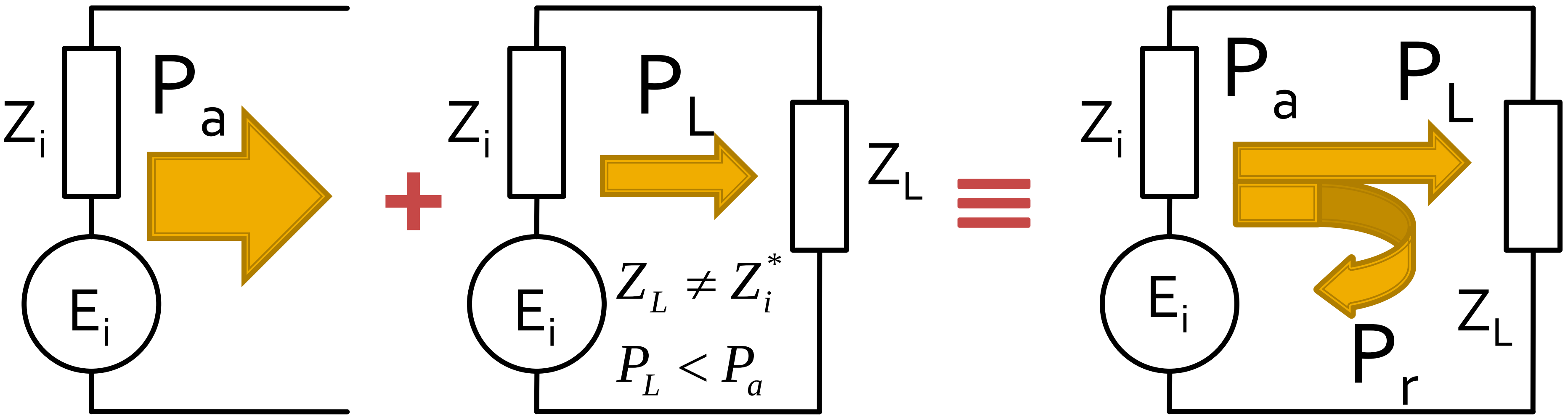
$$P_{avg}=\frac{1}{2}\cdot\mathrm{Re}\left\{V(z)\cdot I(z)^*\right\}=\frac{1}{2}\cdot\frac{\left|V_0^+\right|^2}{Z_0}\cdot\mathrm{Re}\left\{1-\Gamma^*\cdot \underbrace{e^{-2j\cdot\beta\cdot z}+\Gamma\cdot e^{2j\cdot\beta\cdot z}}_{\left(z-z^*\right)=\mathrm{Im}}-\left|\Gamma\right|^2\right\}$$

$$P_{avg}=\frac{1}{2}\cdot\frac{\left|V_0^+\right|^2}{Z_0}\cdot\left(1-\left|\Gamma\right|^2\right)$$

- Total power delivered to the load = Incident power – “Reflected” power
- Return “Loss” [dB]

$$\mathrm{RL}=-20\cdot\log\left|\Gamma\right| \quad [\mathrm{dB}]$$

Reflection and power / Model



- The source has the ability to sent to the load a certain maximum power (available power) P_a
- For a particular load the power sent to the load is less than the maximum (mismatch) $P_L < P_a$
- The phenomenon is **"as if"** (model) some of the power is reflected $P_r = P_a - P_L$
- The power is a **scalar !**

Matching , from the point of view of power transmission

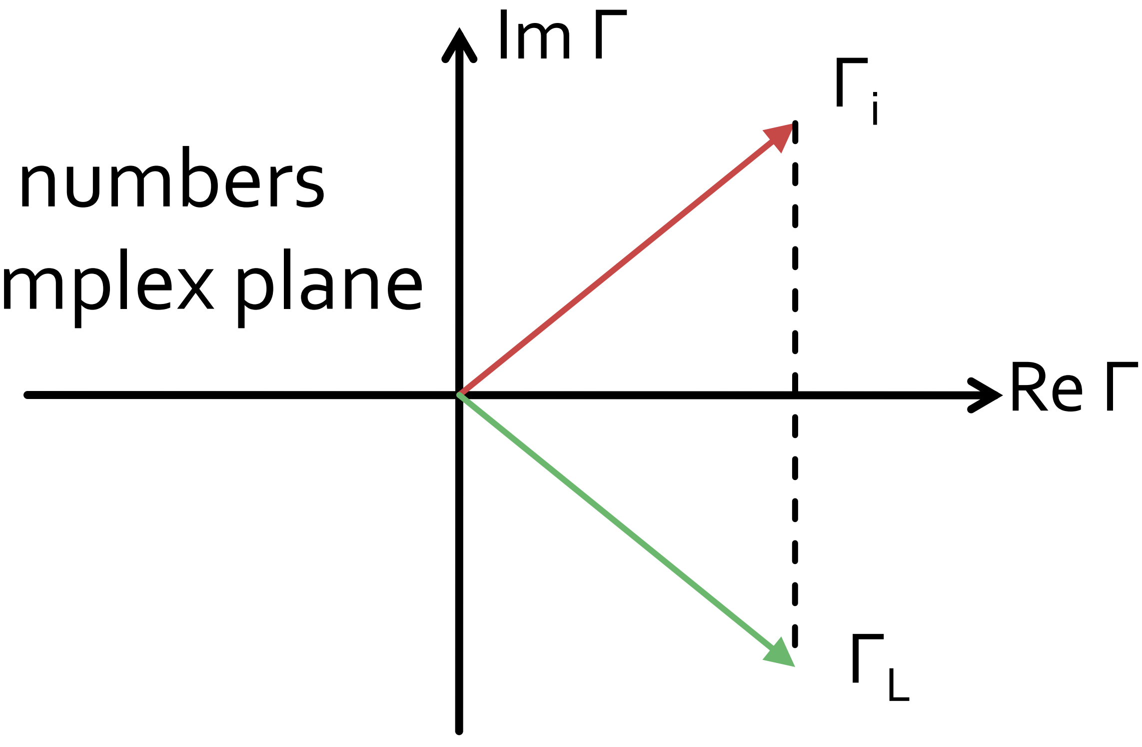
$$\boxed{Z_L = Z_i^*}$$

If we choose a real Zo

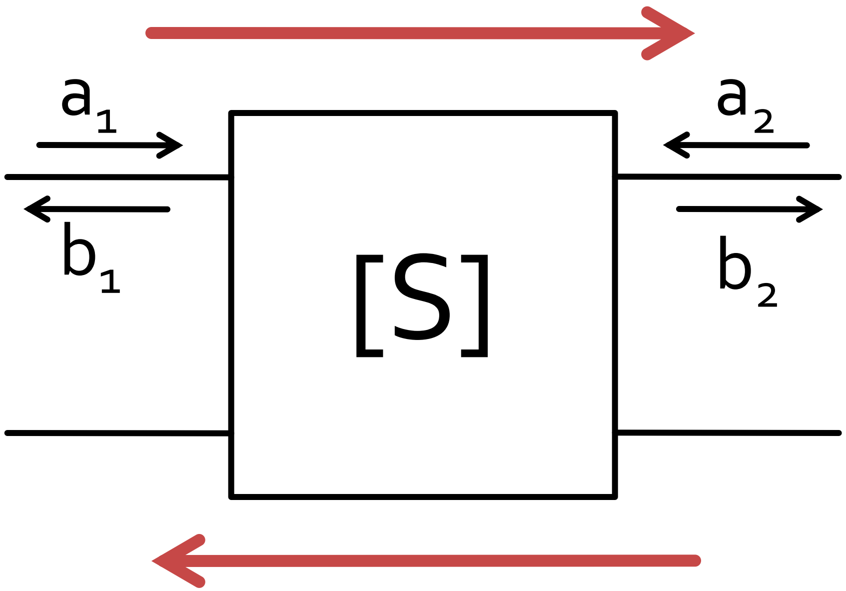
$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

$$\boxed{\Gamma_L = \Gamma_i^*}$$

- complex numbers
- in the complex plane



Scattering matrix – S



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$|S_{21}|^2 = \frac{Power\ in\ Z_0\ load}{Power\ from\ Z_0\ source}$$

- a,b
  - information about signal power **AND** signal phase
- S_{ij}
  - network effect (gain) over signal power **including** phase information

Power dividers and directional couplers

Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~

Introduction

Power dividers and couplers

- Desired functionality:
  - division
  - combining
- of signal power

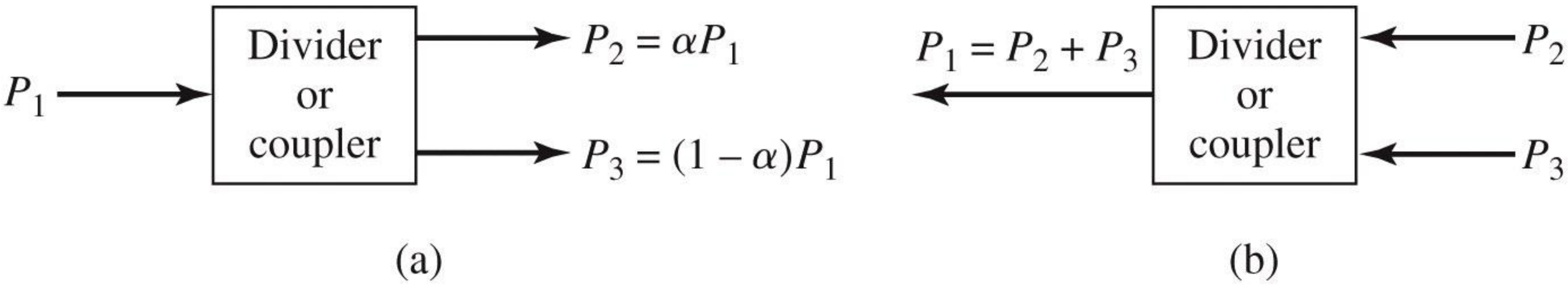
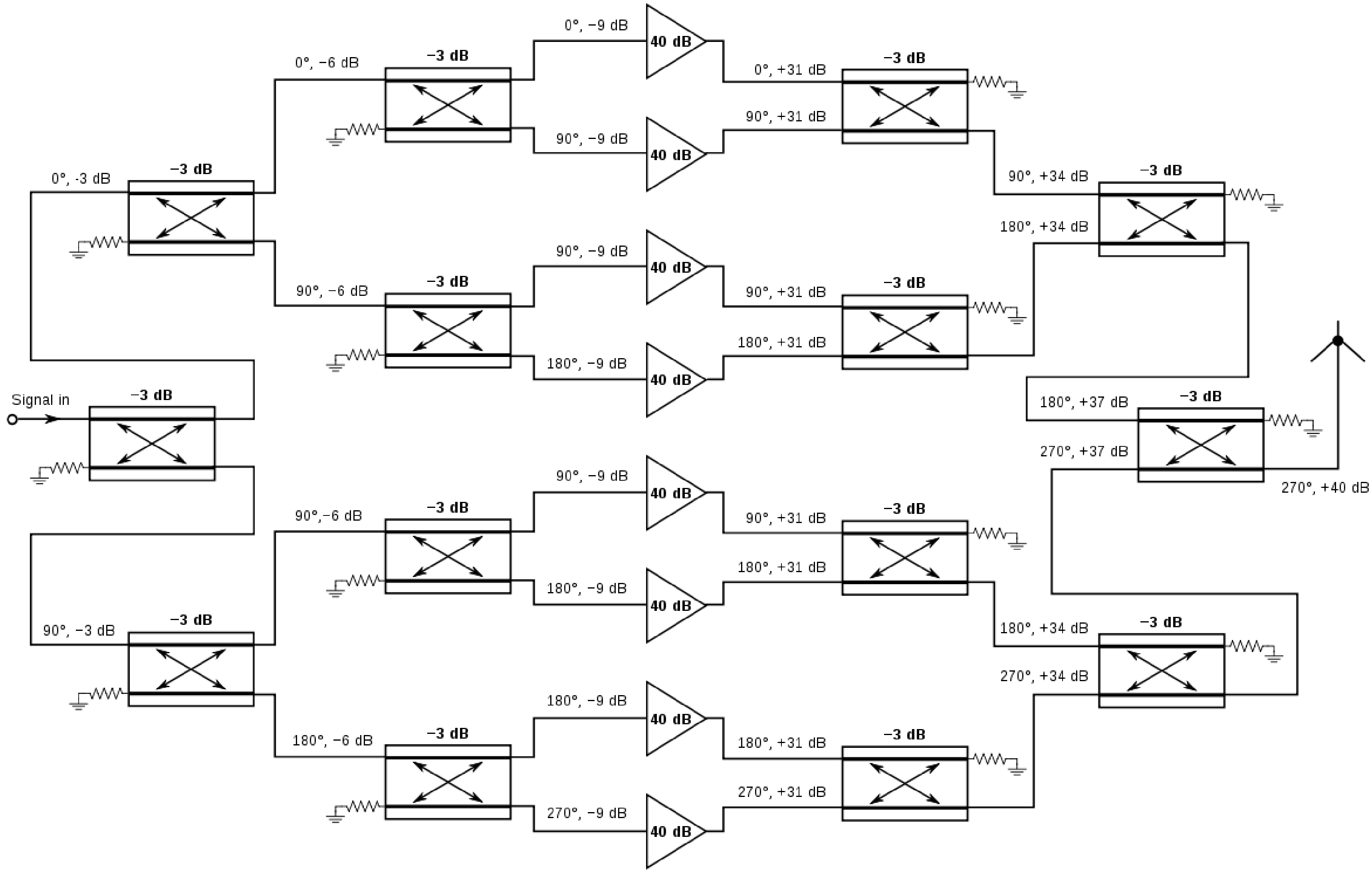


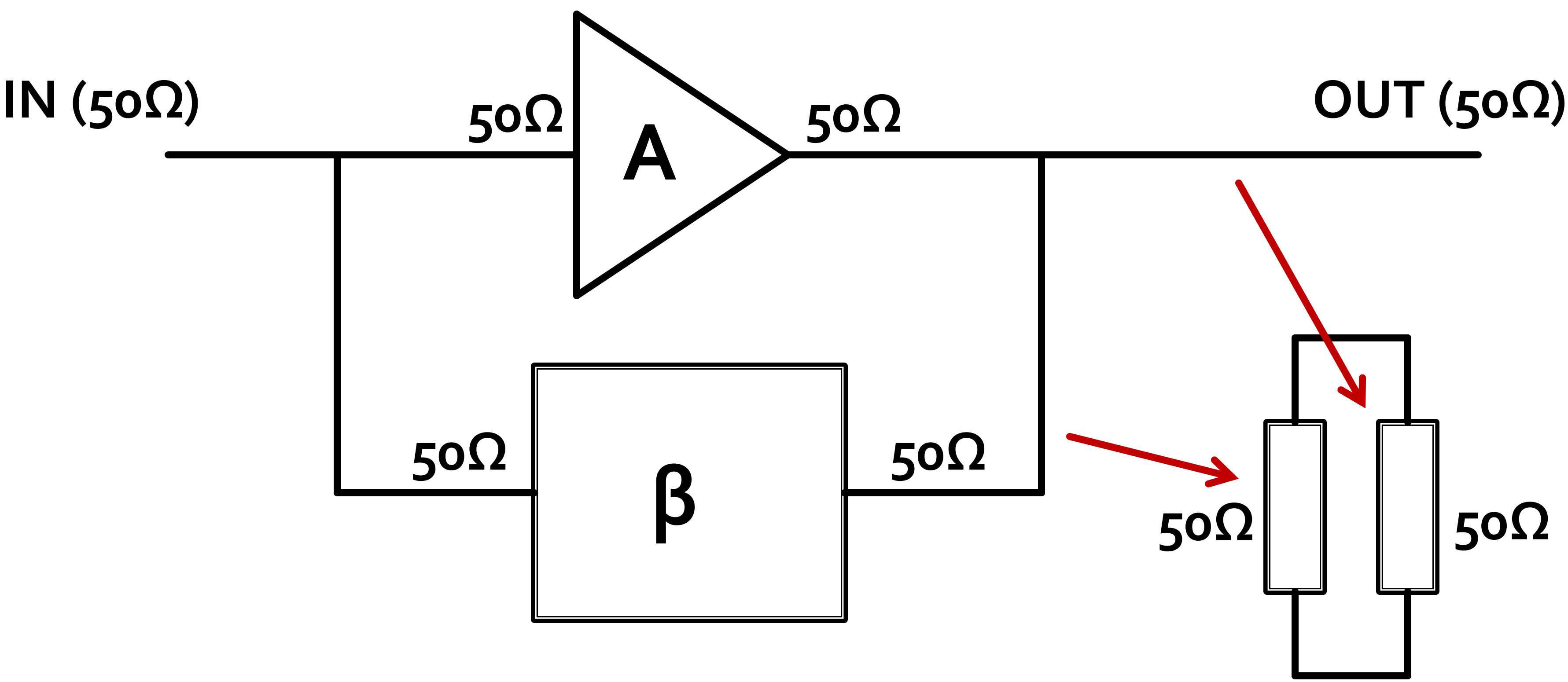
Figure 7.1

Balanced amplifiers



Matching

- feedback amplifier



Three-Port Networks

- also known as T-Junctions
- characterized by a 3x3 **S** matrix

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

- the device is **reciprocal** if it does **not** contain:
  - anisotropic materials (usually ferrites)
  - active circuits
- to avoid power loss, we would like to have a network that is:
  - **lossless**, and
  - **matched at all ports**
    - to avoid reflection power "loss"

Three-Port Networks

- reciprocal

$$[S] = [S]^t \qquad S_{ij} = S_{ji}, \forall j \neq i$$

$$S_{12} = S_{21}, S_{13} = S_{31}, S_{23} = S_{32}$$

- matched at all ports

$$S_{ii} = 0, \forall i \qquad S_{11} = 0, S_{22} = 0, S_{33} = 0$$

- then the S matrix is:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

Three-Port Networks

- reciprocal, matched at all ports, S matrix:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

- lossless network
  - all the power injected in one port will be found exiting the network on all ports

$$[S]^{*}\cdot[S]^{t}=[1] \qquad \sum_{k=1}^{N} S_{ki}\cdot S_{kj}^{*}=\delta_{ij},\forall i,j$$

$$\sum_{k=1}^{N} S_{ki}\cdot S_{ki}^{*}=1 \qquad \sum_{k=1}^{N} S_{ki}\cdot S_{kj}^{*}=0,\forall i\neq j$$

Three-Port Networks

- lossless network

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

$$\sum_{k=1}^{N} S_{ki}\cdot S_{ki}^{*}=1$$

$$\sum_{k=1}^{N} S_{ki}\cdot S_{kj}^{*}=0, \forall i\neq j$$

- 6 equations / 3 unknowns

$$|S_{12}|^2+|S_{13}|^2=1 \qquad S_{13}^{*}S_{23}=0$$

$$|S_{12}|^2+|S_{23}|^2=1 \qquad S_{12}^{*}S_{13}=0$$

$$|S_{13}|^2+|S_{23}|^2=1 \qquad S_{23}^{*}S_{12}=0$$

- no solution is possible

Three-Port Networks

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

- 6 equations / 3 unknowns
  - no solution is possible
- A three-port network **cannot** be simultaneously:
  - reciprocal
  - lossless
  - matched at all ports
- If any one of these three conditions is relaxed, then a physically realizable device is possible

Nonreciprocal Three-Port Networks

- usually containing anisotropic materials, ferrites
- **nonreciprocal**, but matched at all ports and lossless $S_{ij} \neq S_{ji}$
- S matrix

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

- 6 equations / 6 unknowns

$$|S_{12}|^2 + |S_{13}|^2 = 1 \qquad S_{31}^* S_{32} = 0$$
$$|S_{21}|^2 + |S_{23}|^2 = 1 \qquad S_{21}^* S_{23} = 0$$
$$|S_{31}|^2 + |S_{32}|^2 = 1 \qquad S_{12}^* S_{13} = 0$$

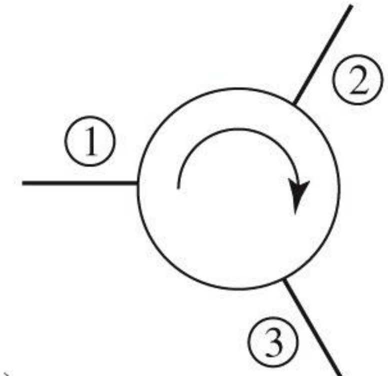
Nonreciprocal Three-Port Networks

- two possible solutions
- circulators
  - clockwise circulation

$$S_{12} = S_{23} = S_{31} = 0$$

$$|S_{21}| = |S_{32}| = |S_{13}| = 1$$

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



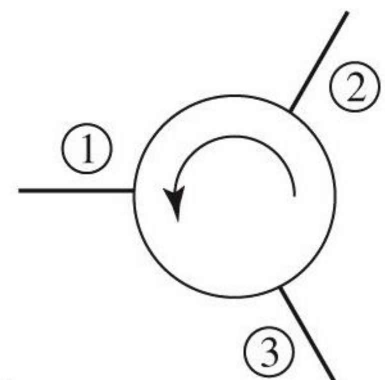
(a)

  - counterclockwise circulation

$$S_{21} = S_{32} = S_{13} = 0$$

$$|S_{12}| = |S_{23}| = |S_{31}| = 1$$

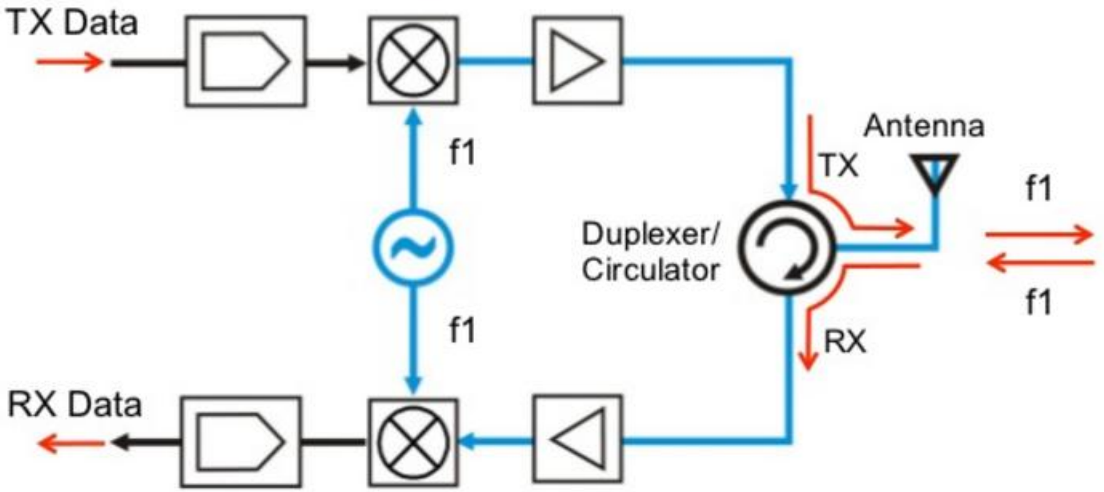
$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

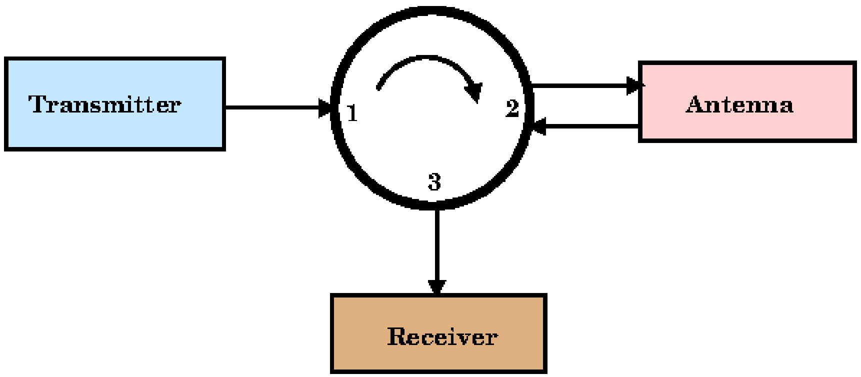


(b)

Nonreciprocal Three-Port Networks

- circulator often found in duplexer





Mismatched Three-Port Networks

- A lossless and reciprocal three-port network can be matched only on two ports, eg. 1 and 2:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

$$S_{13}^* S_{23} = 0$$

$$S_{12}^* S_{13} + S_{23}^* S_{33} = 0$$

$$S_{23}^* S_{12} + S_{33}^* S_{13} = 0$$

$$|S_{12}|^2 + |S_{13}|^2 = 1$$

$$|S_{12}|^2 + |S_{23}|^2 = 1$$

$$|S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 = 1$$

$$S_{13} = S_{23} = 0$$

$$|S_{13}| = |S_{23}|$$

$$|S_{12}| = |S_{33}| = 1$$

Mismatched Three-Port Networks

- A lossless and reciprocal three-port network

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

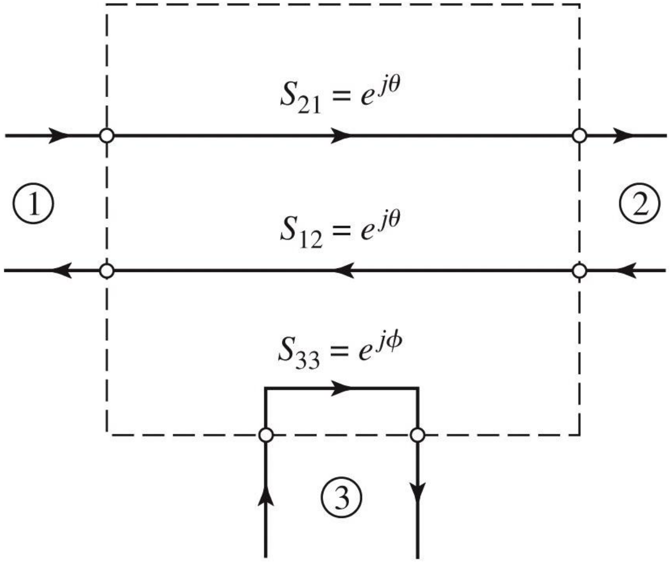
$$S_{13}=S_{23}=0$$

$$|S_{12}|=|S_{33}|=1$$

$$S_{12}=e^{j\theta}$$

$$S_{33}=e^{j\varphi}$$

$$[S]=\begin{bmatrix} 0 & e^{j\theta} & 0 \\ e^{j\theta} & 0 & 0 \\ 0 & 0 & e^{j\phi} \end{bmatrix}$$



- A lossless and reciprocal three-port network **degenerates** into two separate components:
  - a matched two-port **line**
  - a totally **mismatched one-port**:

Four-Port Networks

- characterized by a 4x4 **S** matrix

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

- the device is **reciprocal** if it does **not** contain:
  - anisotropic materials (usually ferrites)
  - active circuits
- to avoid power loss, we would like to have a network that is:
  - **lossless**, and
  - **matched at all ports**
    - to avoid reflection power "loss"

Four-Port Networks

- reciprocal

$$[S]=[S]^t \qquad S_{ij}=S_{ji}, \forall j \neq i$$

$$S_{12}=S_{21}, S_{13}=S_{31}, S_{23}=S_{32}$$

- matched at all ports

$$S_{ii}=0, \forall i \qquad S_{11}=0, S_{22}=0, S_{33}=0, S_{44}=0$$

- then the S matrix is:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

Four-Port Networks

- reciprocal, matched at all ports, S matrix:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

- lossless network
  - all the power injected in one port will be found exiting the network on all ports

$$[S]^* \cdot [S]^t = [1] \qquad \sum_{k=1}^{N} S_{ki} \cdot S_{kj}^* = \delta_{ij}, \forall i, j$$

$$\sum_{k=1}^{N} S_{ki} \cdot S_{ki}^* = 1 \qquad \sum_{k=1}^{N} S_{ki} \cdot S_{kj}^* = 0, \forall i \neq j$$

Four-Port Networks

$$S_{13}^* \cdot S_{23} + S_{14}^* \cdot S_{24} = 0 \quad / \cdot S_{24}^*$$
$$S_{14}^* \cdot S_{13} + S_{24}^* \cdot S_{23} = 0 \quad / \cdot S_{13}^*$$
$$S_{14}^* \cdot \left(|S_{13}|^2 - |S_{24}|^2 \right) = 0$$

$$S_{12}^* \cdot S_{23} + S_{14}^* \cdot S_{34} = 0 \quad / \cdot S_{12}$$
$$S_{14}^* \cdot S_{12} + S_{34}^* \cdot S_{23} = 0 \quad / \cdot S_{34}^*$$
$$S_{23} \cdot \left(|S_{12}|^2 - |S_{34}|^2 \right) = 0$$

- one solution: $S_{14} = S_{23} = 0$
- resulting coupler is **directional**

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix}$$

$$|S_{12}|^2 + |S_{13}|^2 = 1$$
$$|S_{12}|^2 + |S_{24}|^2 = 1$$
$$|S_{13}|^2 + |S_{34}|^2 = 1$$
$$|S_{24}|^2 + |S_{34}|^2 = 1$$

$$|S_{13}| = |S_{24}|$$

$$|S_{12}| = |S_{34}|$$

Four-Port Networks

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix}$$

$$|S_{12}|=|S_{34}|=\alpha \qquad |S_{13}|=|S_{24}|=\beta$$

β – voltage coupling coefficient

- We can choose the phase reference

$$S_{12}=S_{34}=\alpha \qquad S_{13}=\beta\cdot e^{j\theta} \qquad S_{24}=\beta\cdot e^{j\phi}$$

$$S_{12}^{*}\cdot S_{13}+S_{24}^{*}\cdot S_{34}=0 \quad \rightarrow \quad \theta+\phi=\pi\pm 2\cdot n\cdot\pi$$

$$|S_{12}|^{2}+|S_{24}|^{2}=1 \quad \rightarrow \quad \alpha^{2}+\beta^{2}=1$$

- The other possible solution for previous equations offer either essentially the same result (with a different phase reference) or the degenerate case (2 separate two port networks side by side)

$$S_{14}^{*}\cdot\left(|S_{13}|^{2}-|S_{24}|^{2}\right)=0 \qquad S_{23}\cdot\left(|S_{12}|^{2}-|S_{34}|^{2}\right)=0$$

Four-Port Networks

- A four-port network simultaneously:
  - matched at all ports
  - reciprocal
  - lossless
- is **always directional**
  - the signal power injected into one port is transmitted **only towards two** of the other three ports

$$[S]=\begin{bmatrix} 0 & \alpha & \beta\cdot e^{j\theta} & 0 \\ \alpha & 0 & 0 & \beta\cdot e^{j\phi} \\ \beta\cdot e^{j\theta} & 0 & 0 & \alpha \\ 0 & \beta\cdot e^{j\phi} & \alpha & 0 \end{bmatrix}$$

Four-Port Networks

- two particular choices commonly occur in practice
  - A Symmetric Coupler $\theta = \phi = \pi/2$

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

  - An Antisymmetric Coupler $\theta = 0, \phi = \pi$

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

Directional Coupler

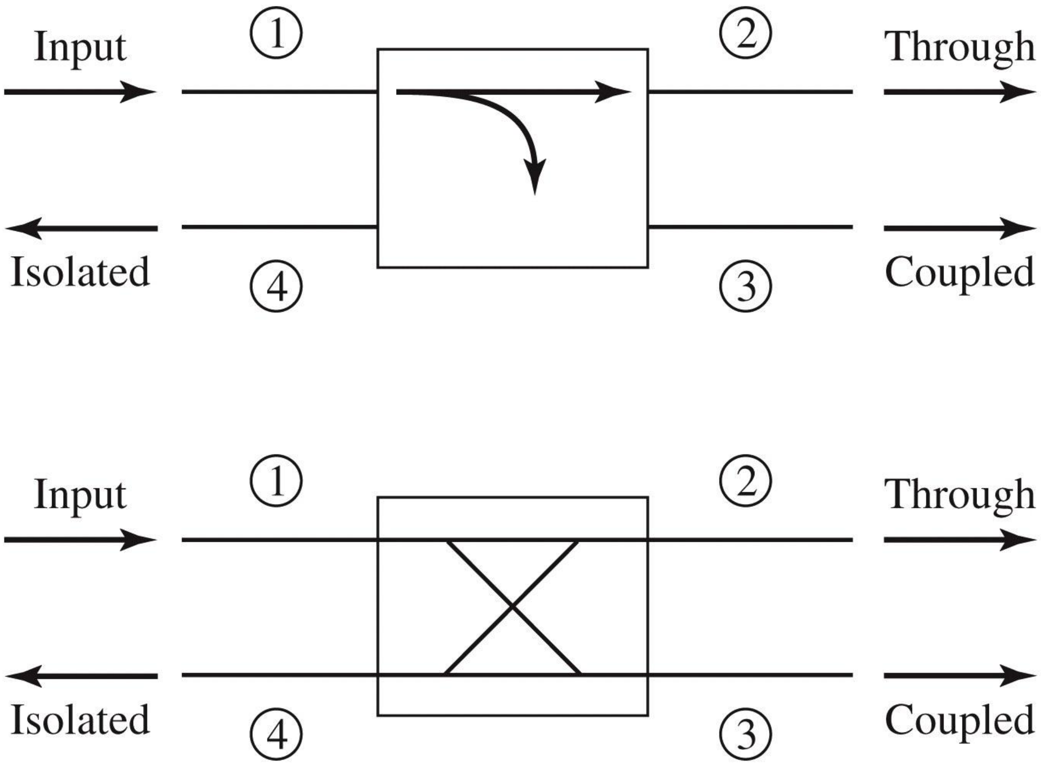


Figure 7.4

$$|S_{12}|^2 = \alpha^2 = 1 - \beta^2$$

$$|S_{13}|^2 = \beta^2$$

Coupling

$$C = 10\log\frac{P_1}{P_3} = -20\cdot\log(\beta)\,[\mathrm{dB}]$$

Directivity

$$D = 10\log\frac{P_3}{P_4} = 20\cdot\log\left(\frac{\beta}{|S_{14}|}\right)[\mathrm{dB}]$$

Isolation

$$I = 10\log\frac{P_1}{P_4} = -20\cdot\log|S_{14}|\,[\mathrm{dB}]$$

$$I = D + C\,,\ \ [dB]$$

Power dividers

Three-Port Networks

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

- 6 equations / 3 unknowns
  - no solution is possible
- A three-port network **cannot** be simultaneously:
  - reciprocal
  - **lossless**
  - matched at all ports
- If any one of these three conditions is relaxed, then a physically realizable device is possible

Power division of the T-junction

- consists in splitting an input line into two separate output lines
- available in various technologies for the lines

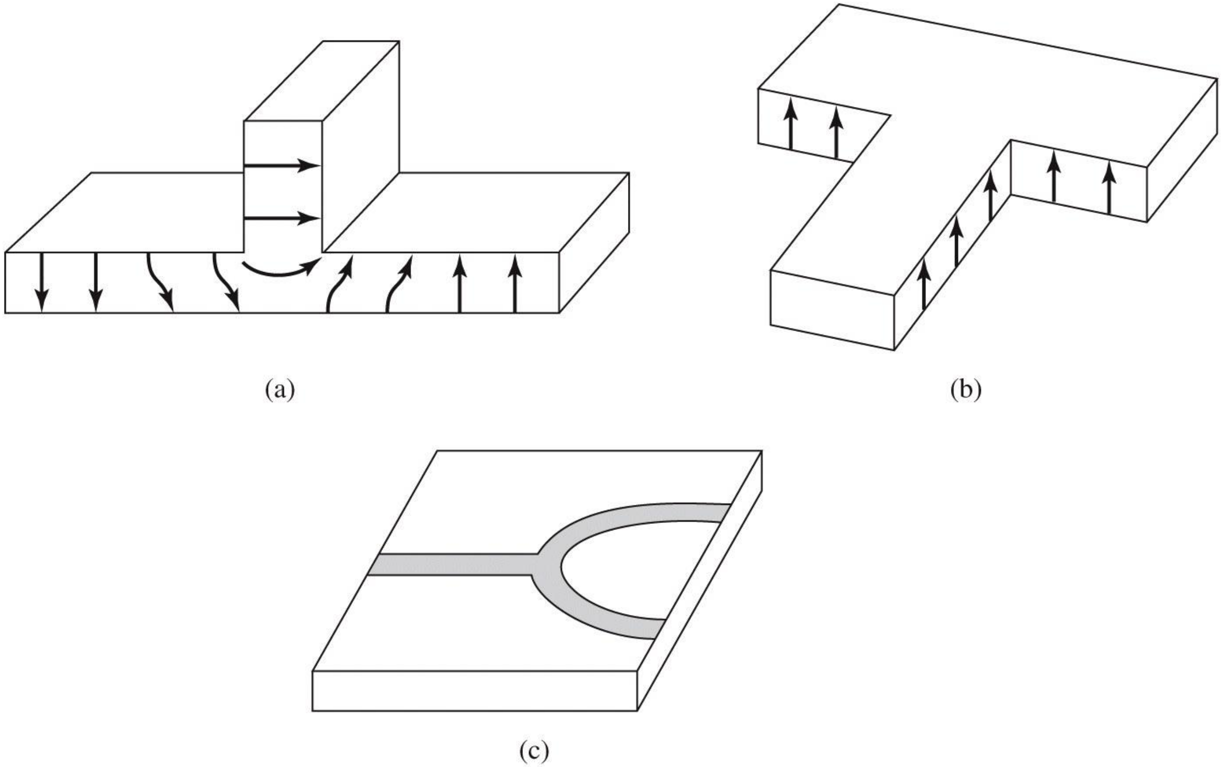


Figure 7.5

Power division of the T-junction

- if the lines are lossless, the network is reciprocal, so it cannot be matched at all ports simultaneously

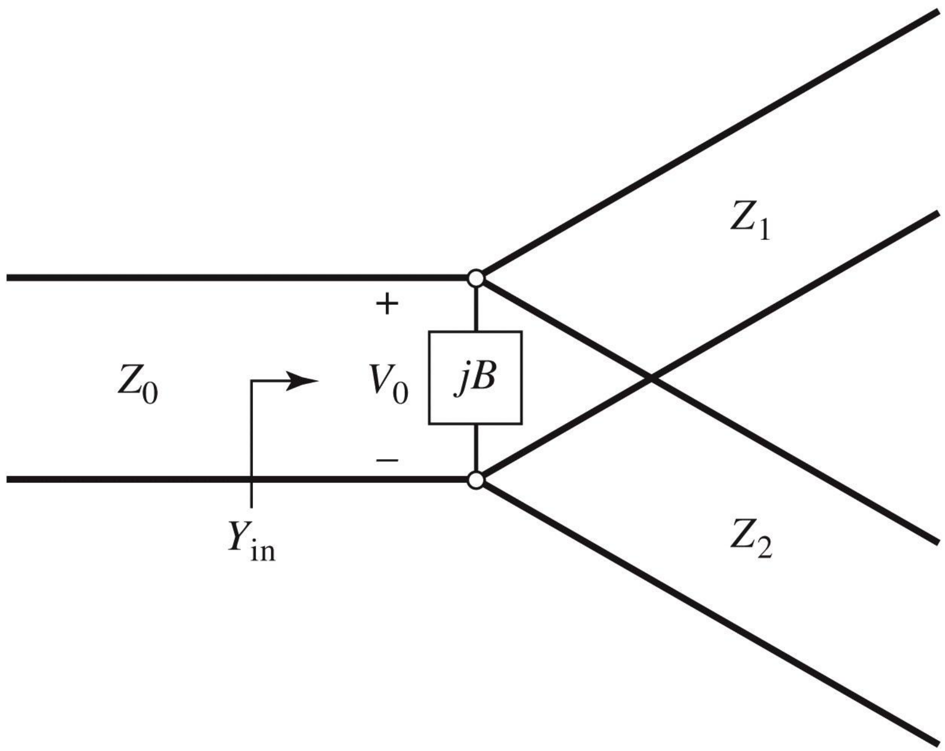


Figure 7.6

- there may be fringing fields and higher order modes associated with the discontinuity at such a junction
- the stored energy can be accounted for by a lumped susceptance: **B**
- Designing the power divider targets matching to the input line Z_o
  - outputs (unmatched, Z_1 and Z_2) can be, if needed, matched to Z_o ($\lambda/4$, binomial, Chebyshev)

Power division of the T-junction

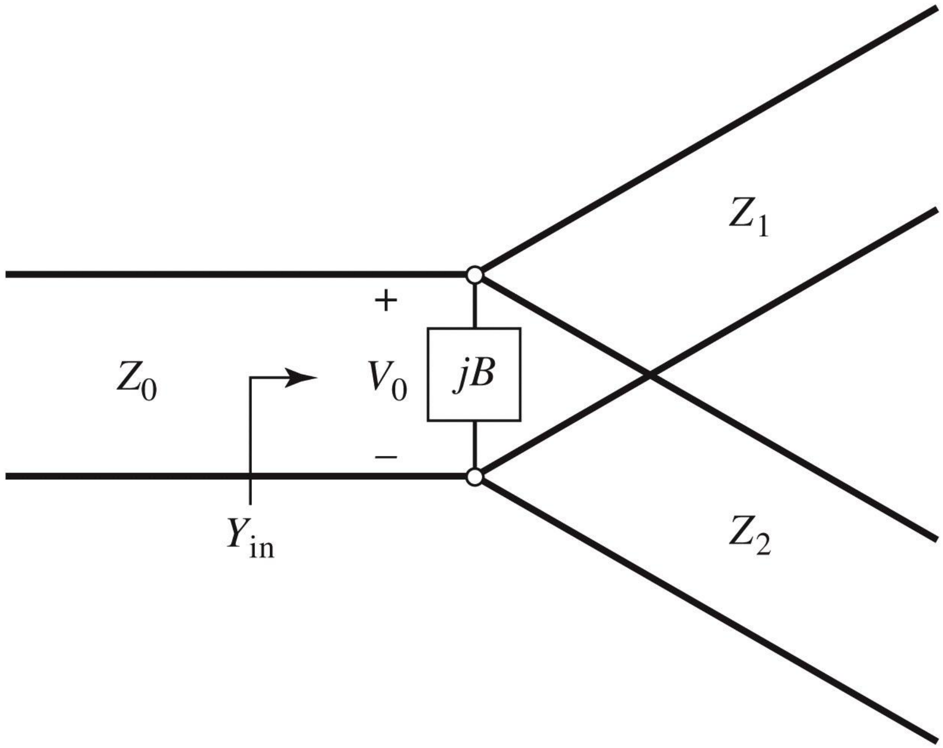


Figure 7.6

$$Y_{in} = j \cdot B + \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}$$

- If the transmission lines are assumed to be lossless, then the characteristic impedances are real
- the matching condition can be met only if **B ≅ 0** thus the matching condition is:

$$\frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}$$

In practice, if **B** is not negligible, some type of discontinuity compensation or a reactive tuning element can usually be used to cancel this susceptance, at least over a narrow frequency range.

Power division of the T-junction

- if V_o is the voltage at the junction, we can compute how the input power is divided between the two output lines

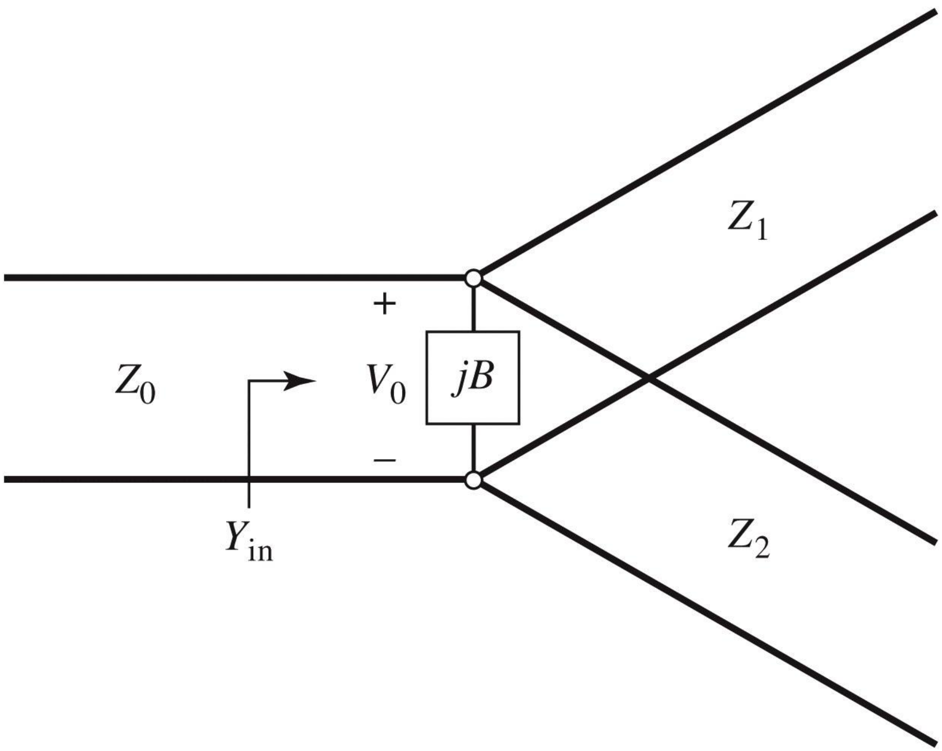


Figure 7.6

$$P_{in} = \frac{1}{2} \cdot \frac{V_0^2}{Z_0}$$

$$P_1 = \frac{1}{2} \cdot \frac{V_0^2}{Z_1}$$

$$P_2 = \frac{1}{2} \cdot \frac{V_0^2}{Z_2}$$

then:

$$P_{in} = P_1 + P_2$$ (lossless/input matching)

$$\frac{P_1}{P_2} = \frac{Z_2}{Z_1} = \alpha$$ (power division between the two output lines)

$$P_1 = P_{in} \cdot \frac{Z_2}{Z_1 + Z_2} \qquad P_2 = P_{in} \cdot \frac{Z_1}{Z_1 + Z_2}$$

$$P_1 = P_{in} \cdot \frac{\alpha}{1+\alpha} \qquad P_2 = P_{in} \cdot \frac{1}{1+\alpha}$$

$$Z_1 = Z_0 \cdot \left(1 + \frac{1}{\alpha}\right) \qquad Z_2 = Z_0 \cdot (1+\alpha)$$

Power division of the T-junction

- S matrix
  - lossless (unitary matrix)
  - reciprocal (symmetrical matrix)
  - input port is matched $S_{11} = 0$

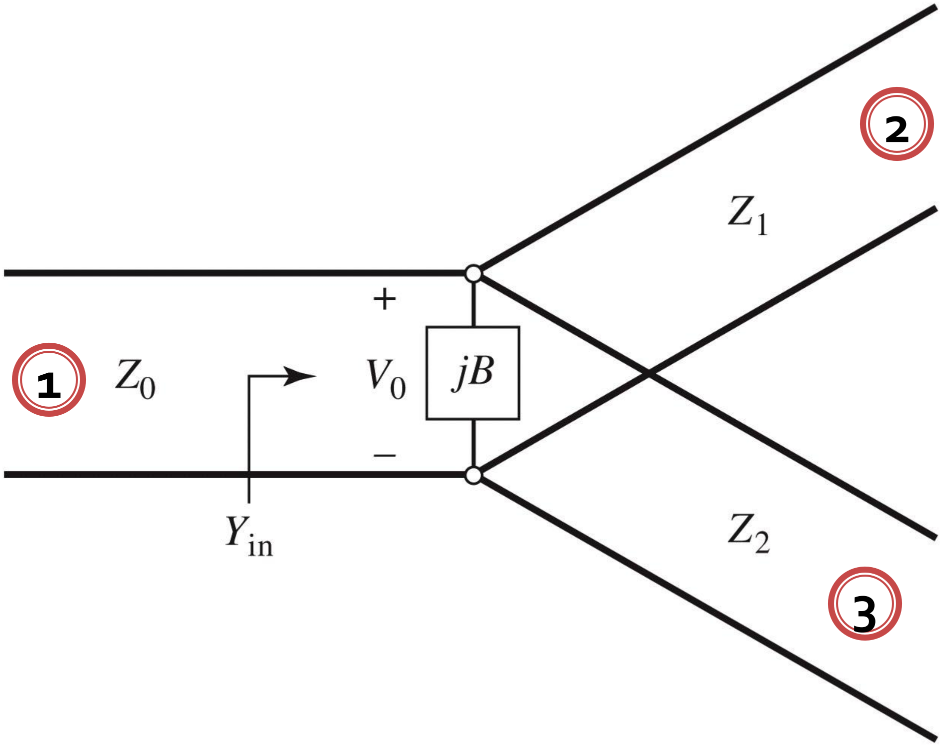


Figure 7.6

$$P_2 = P_1 \cdot \frac{\alpha}{1+\alpha} \qquad S_{21} = S_{12} = \sqrt{\frac{\alpha}{1+\alpha}}$$

$$P_3 = P_1 \cdot \frac{1}{1+\alpha} \qquad S_{31} = S_{13} = \sqrt{\frac{1}{1+\alpha}}$$

the reflection coefficients seen looking into the output ports

$$S_{22} = \Gamma_1 = \frac{Z_0 \,||\, Z_2 - Z_1}{Z_0 \,||\, Z_2 + Z_1} = -\frac{1}{1+\alpha}$$

$$S_{33} = \Gamma_2 = \frac{Z_0 \,||\, Z_1 - Z_2}{Z_0 \,||\, Z_1 + Z_2} = -\frac{\alpha}{1+\alpha}$$

Power division of the T-junction

$$[S]=\begin{bmatrix} 0 & \sqrt{\dfrac{\alpha}{1+\alpha}} & \sqrt{\dfrac{1}{1+\alpha}} \\ \sqrt{\dfrac{\alpha}{1+\alpha}} & -\dfrac{1}{1+\alpha} & x \\ \sqrt{\dfrac{1}{1+\alpha}} & x & -\dfrac{\alpha}{1+\alpha} \end{bmatrix}$$

Unitary matrix, columns 1 and 2

$$0-\frac{1}{1+\alpha}\cdot\sqrt{\frac{\alpha}{1+\alpha}}+x\cdot\sqrt{\frac{1}{1+\alpha}}=0$$

$$S_{23}=S_{32}=\frac{\sqrt{\alpha}}{1+\alpha}$$

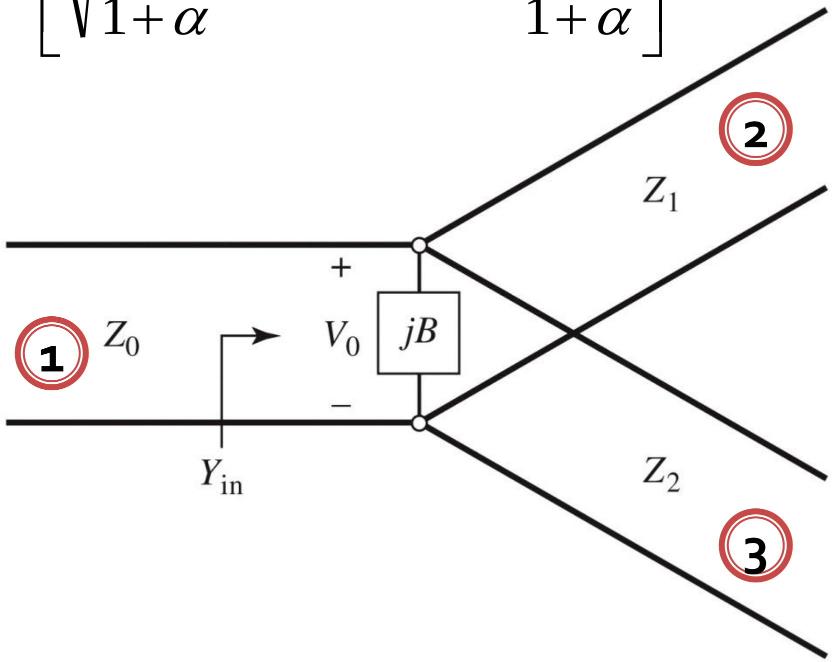


Figure 7.6

$$[S]=\begin{bmatrix} 0 & \sqrt{\dfrac{\alpha}{1+\alpha}} & \sqrt{\dfrac{1}{1+\alpha}} \\ \sqrt{\dfrac{\alpha}{1+\alpha}} & -\dfrac{1}{1+\alpha} & \dfrac{\sqrt{\alpha}}{1+\alpha} \\ \sqrt{\dfrac{1}{1+\alpha}} & \dfrac{\sqrt{\alpha}}{1+\alpha} & -\dfrac{\alpha}{1+\alpha} \end{bmatrix}$$

Power division of the T-junction

- 3dB divider
  - equal splitting of the power between the two outputs
  - $Z_1 = Z_2 = 2 \cdot Z_0$, $\alpha = 1$

$$[S]=\begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

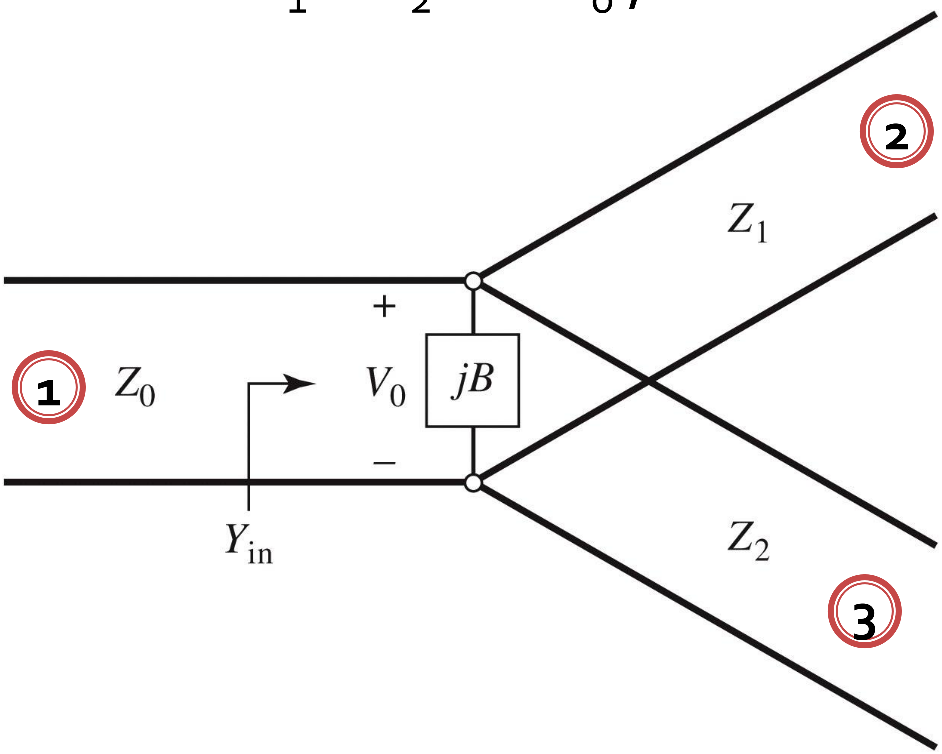


Figure 7.6

If we add λ/4 transformers to match outputs to Z_0 S matrix:

$$[S]=\begin{bmatrix} 0 & -\frac{j}{\sqrt{2}} & -\frac{j}{\sqrt{2}} \\ -\frac{j}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{j}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Example

- Design a lossless T-junction divider with a 30Ω source impedance to give a 3:1 power split. Design quarter-wave matching transformers to convert the impedances of the output lines to 30Ω. (**Pozar** problem)

$$P_{in} = \frac{1}{2} \cdot \frac{V_0^2}{Z_0} \qquad \begin{cases} P_1 + P_2 = P_{in} \\ P_1 : P_2 = 3:1 \end{cases} \Rightarrow \begin{cases} P_1 = \frac{1}{4} \cdot P_{in} \\ P_2 = \frac{3}{4} \cdot P_{in} \end{cases}$$

$$P_1 = \frac{1}{2} \cdot \frac{V_0^2}{Z_1} = \frac{1}{4} \cdot P_{in} \qquad Z_1 = 4 \cdot Z_0 = 120\ \Omega$$

Input match check

$$Z_{in} = 40\ \Omega \,\|\, 120\ \Omega = 30\ \Omega$$

$$P_2 = \frac{1}{2} \cdot \frac{V_0^2}{Z_2} = \frac{3}{4} \cdot P_{in} \qquad Z_2 = 4 \cdot Z_0 / 3 = 40\ \Omega$$

quarter-wave transformers $Z_c^i = \sqrt{Z_i \cdot Z_L}$

$$Z_c^1 = \sqrt{Z_1 \cdot Z_L} = \sqrt{120\Omega \cdot 30\Omega} = 60\Omega \qquad Z_c^2 = \sqrt{Z_2 \cdot Z_L} = \sqrt{40\Omega \cdot 30\Omega} = 34.64\Omega$$

Resistive Divider

- If a three-port divider contains lossy components, it can be made to be :
  - reciprocal
  - matched at all ports

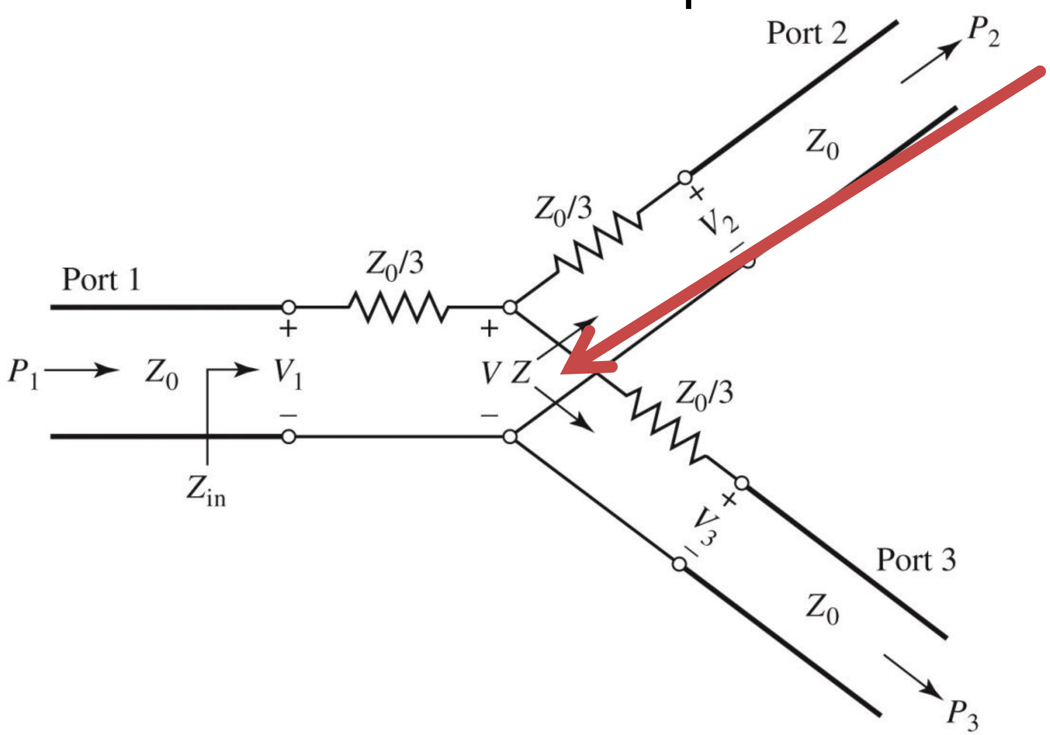


Figure 7.7

The impedance Z, seen looking into the Zo/3 resistor followed by a terminated output line:

$$Z = \frac{Z_0}{3} + Z_0 = \frac{4Z_0}{3}$$

The input line will be terminated with a Zo/3 resistor in series with two such lines Z in parallel

$$Z_{in} = \frac{Z_0}{3} + \frac{1}{2} \cdot \frac{4Z_0}{3} = Z_0$$

so it will be matched: $S_{11} = 0$

from symmetry: $S_{11} = S_{22} = S_{33} = 0$

Resistive Divider

- If a three-port divider contains lossy components, it can be made to be :
  - reciprocal
  - matched at all ports $S_{11} = S_{22} = S_{33} = 0$

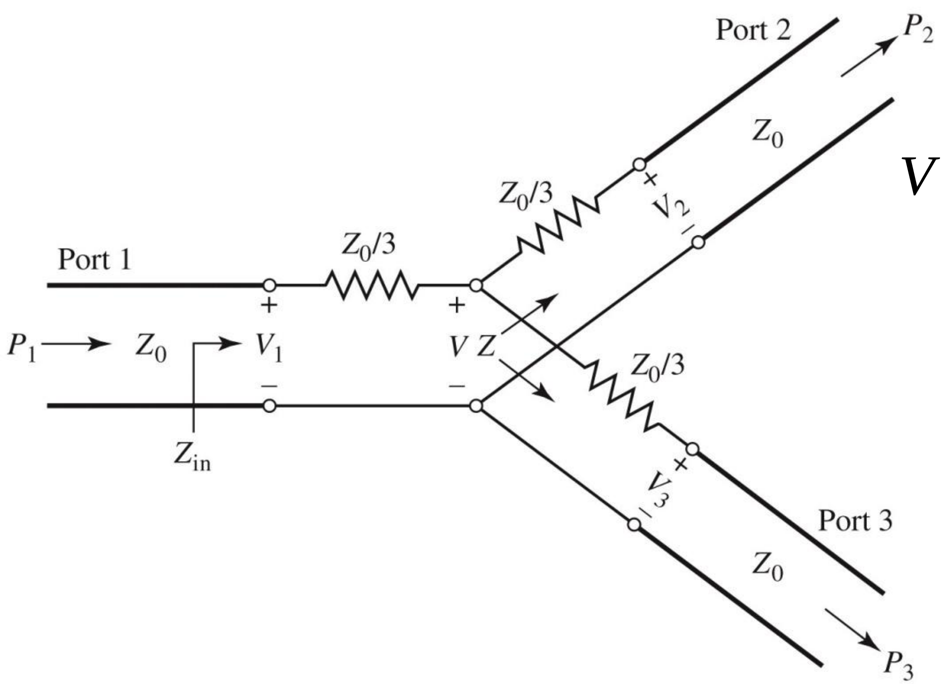


Figure 7.7

If the voltage at port 1 is V1, then by voltage division the voltage V at the junction is:

$$V = V_1 \cdot \frac{Z/2}{Z/2 + Z_0/3} = V_1 \cdot \frac{2Z_0/3}{2Z_0/3 + Z_0/3} = \frac{2}{3} \cdot V_1$$

The output voltages are, again by voltage division :

$$V_2 = V_3 = V \cdot \frac{Z_0}{Z_0 + Z_0/3} = \frac{3}{4} \cdot V = \frac{1}{2} \cdot V_1$$

$$S_{21} = S_{31} = \frac{1}{2}$$

from symmetry: $S_{21} = S_{31} = S_{23} = \frac{1}{2}$

Resistive Divider

- If a three-port divider contains lossy components, it can be made to be :
  - reciprocal (S matrix is symmetrical) $S_{21} = S_{31} = S_{23} = \frac{1}{2}$
  - matched at all ports $S_{11} = S_{22} = S_{33} = 0$

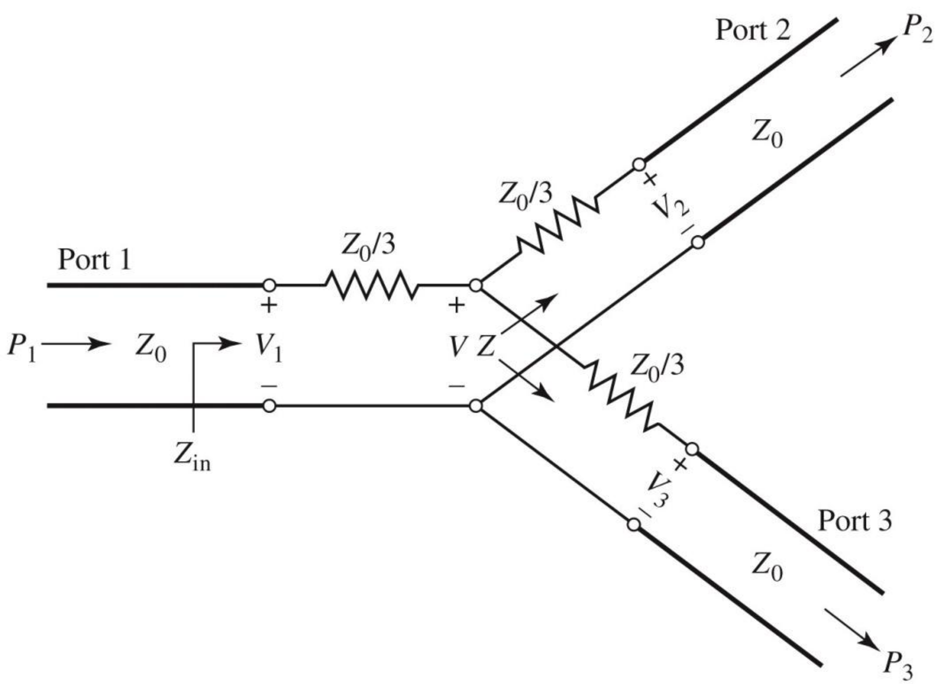


Figure 7.7

S matrix: $$[S] = \frac{1}{2} \cdot \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

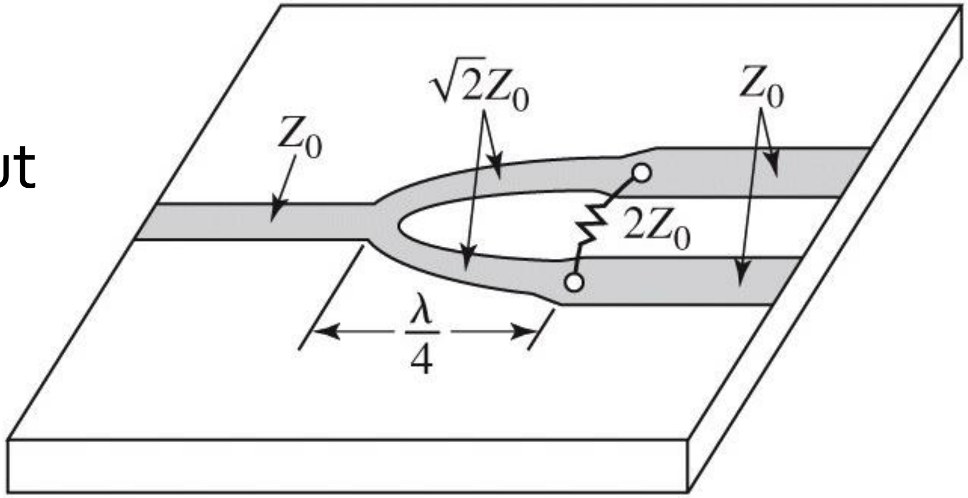
Powers: $$P_{in} = \frac{1}{2} \cdot \frac{V_1^2}{Z_0}$$

$$P_2 = P_3 = \frac{1}{2} \cdot \frac{(1/2 V_1)^2}{Z_0} = \frac{1}{8} \cdot \frac{V_1^2}{Z_0} = \frac{1}{4} \cdot P_{in}$$

Half of the supplied power is dissipated in the 3 resistors. The output powers are 6 dB below the input power level

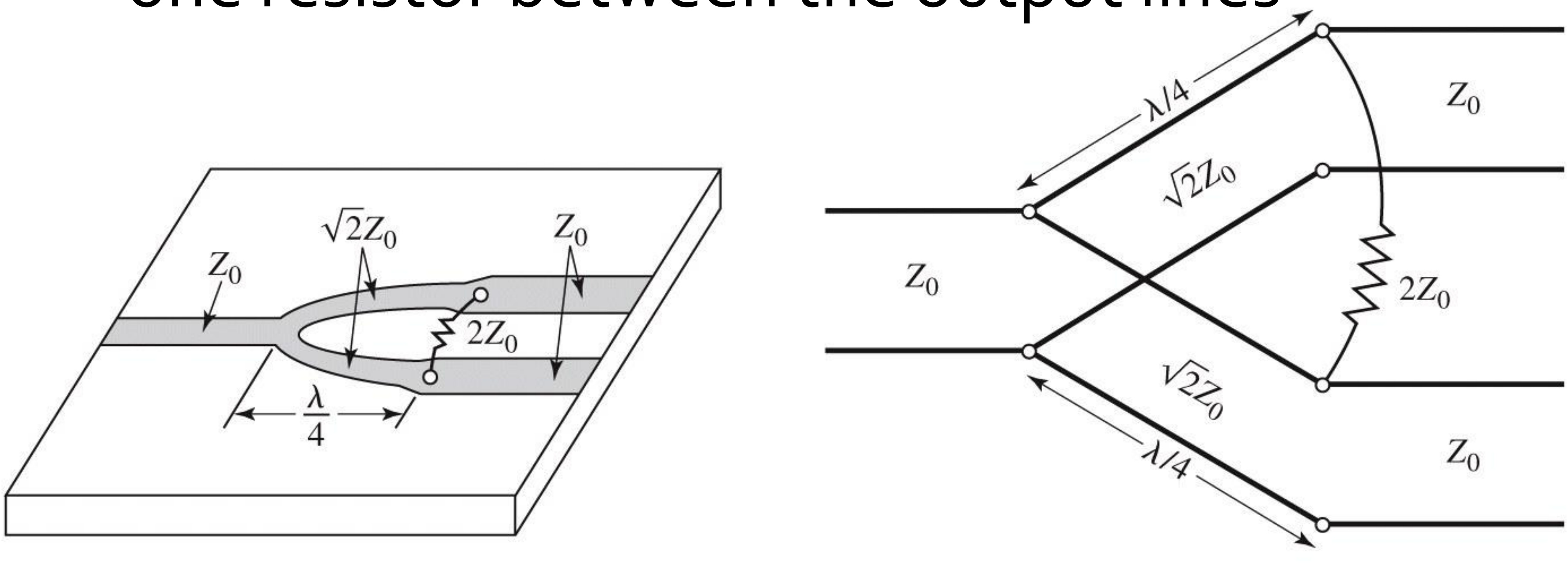
The Wilkinson power divider

- Previous power dividers suffer from a major drawback, there is not isolation between the two output ports $S_{23} = S_{32} \neq 0$
  - this requirement is important in some applications
- The Wilkinson power divider solves this problem
  - it also has the useful property of appearing **lossless** when the output ports are matched
  - only reflected power from the output ports is dissipated



The Wilkinson power divider

- one input line
- two $\lambda/4$ transformers
- one resistor between the output lines



(a) (b)

Figure 7.8

Even/Odd Mode Analysis

- In linear circuits we can use the superposition principle
- advantages
  - reduction of the circuit complexity
  - decrease of the number of ports (**main** advantage)

Response (ODD + EVEN) = Response (ODD) + Response (EVEN)

We can benefit from existing symmetries !!

The Wilkinson power divider

- the circuit in normalized and symmetric form

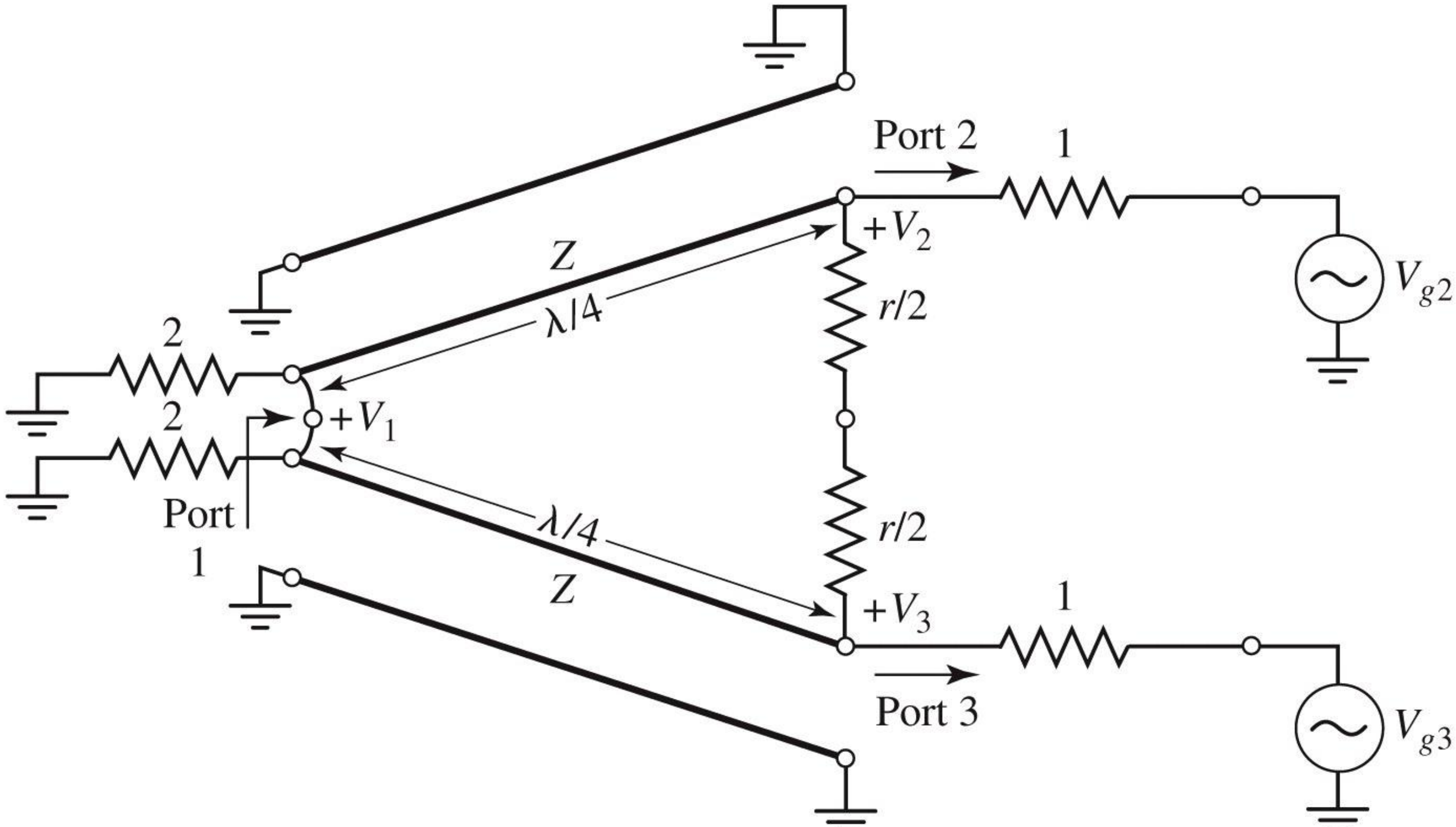
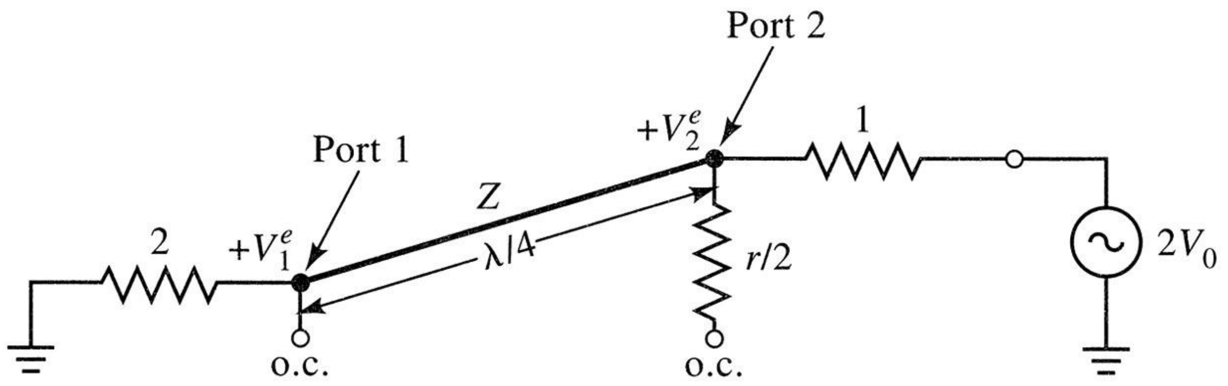


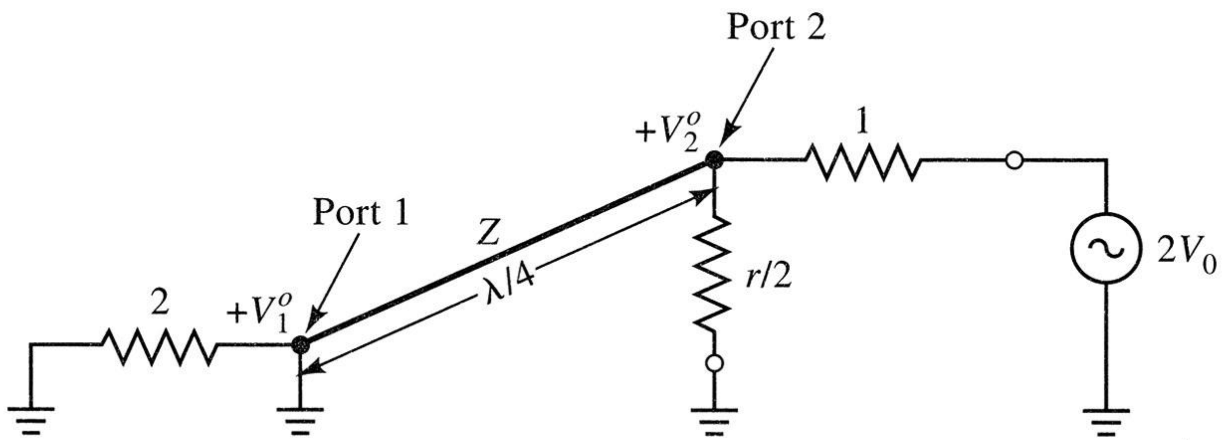
Figure 7.9

The Wilkinson power divider

- Even/Odd Mode Analysis



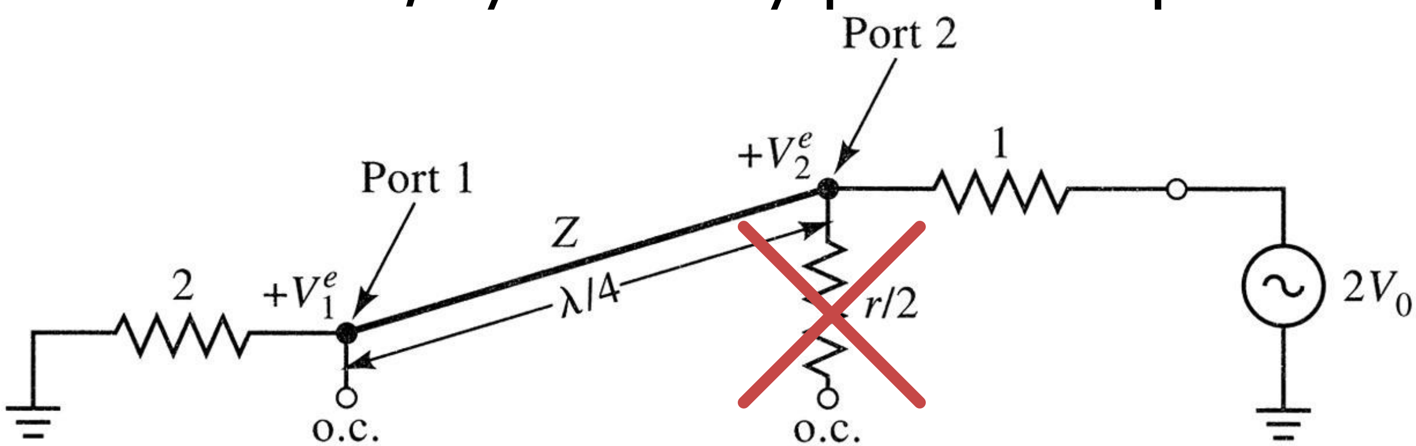
(a)



(b)

The Wilkinson power divider

- even mode, symmetry plane is open circuit



looking into port 2, λ/4 transformer with 2 load $Z_{in2}^e = \frac{Z^2}{2}$ if $Z = \sqrt{2}$ port 2 is matched $Z_{in2}^e = 1$

$$V(x) = V^+ \cdot \left(e^{-j\beta \cdot x} + \Gamma \cdot e^{j\beta \cdot x}\right)$$

x=0 at port 1

x=-λ/4 at port 2

$$V_2^e = V(-\lambda/4) = jV^+ \cdot (1 - \Gamma) = V_0$$

$$Z_{in2}^e = 1$$

$$V_1^e = V(0) = V^+ \cdot (1+\Gamma) = jV_0 \cdot \frac{\Gamma + 1}{\Gamma - 1}$$

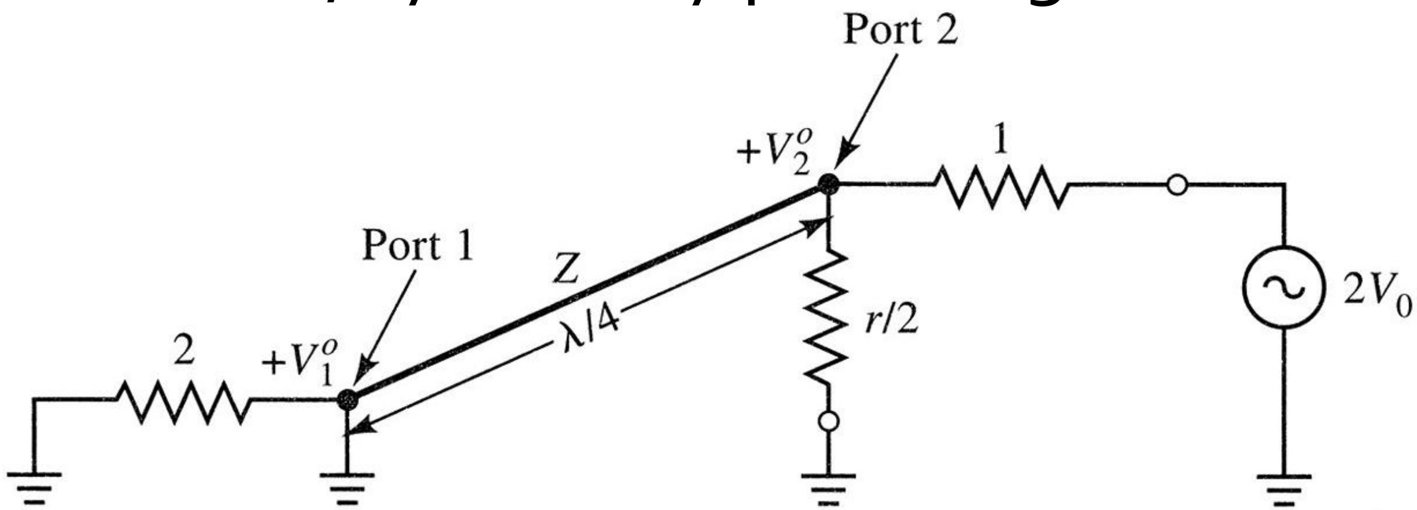
Γ: reflection coefficient seen at port 1 looking toward the resistor of normalized value 2 from the transformer $Z = \sqrt{2}$

$$\Gamma = \frac{2-\sqrt{2}}{2+\sqrt{2}}$$

$$V_1^e = -jV_0\sqrt{2}$$

The Wilkinson power divider

- odd mode, symmetry plane is grounded



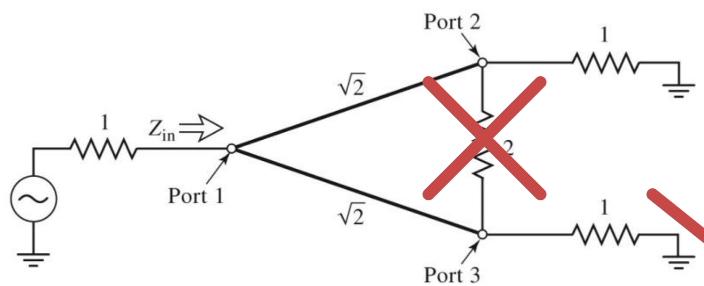
looking from port 2 the λ/4 line is short-circuited, impedance seen from port 2 is ∞ $\quad Z_{in2}^o = r/2 \quad$ if $\quad r = 2 \quad$ port 2 is matched

$$Z_{in2}^o = 1 \rightarrow V_2^o = V_0$$

$V_1^o = 0 \quad$ in the odd mode all the power is dissipated in the *r*/2 resistor

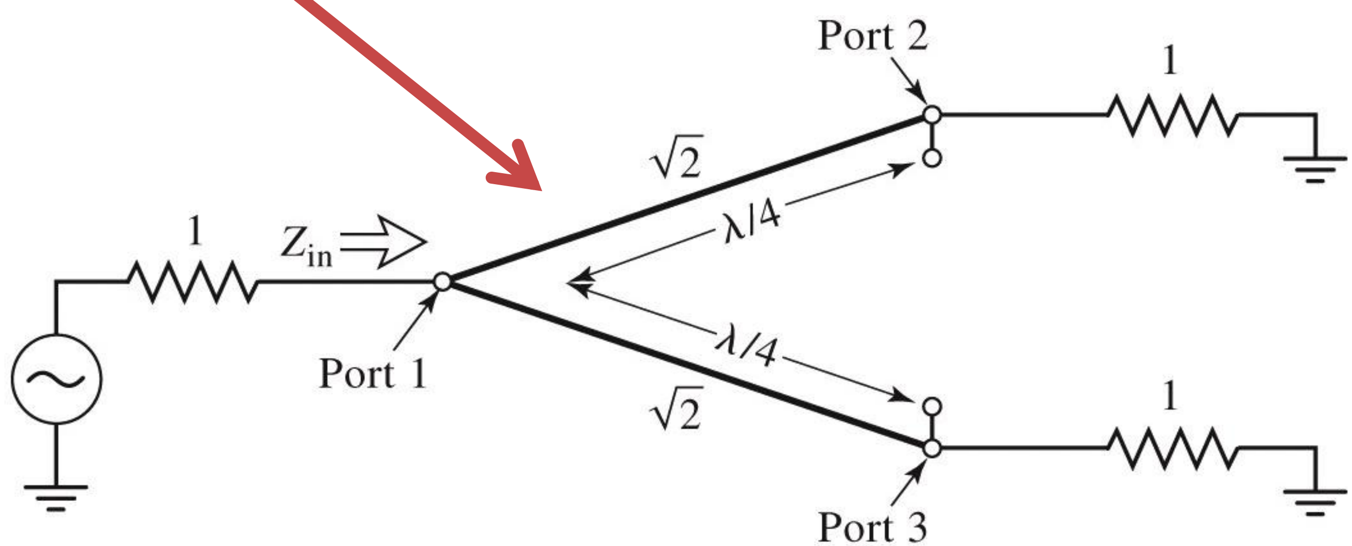
The Wilkinson power divider

- input impedance in port 1



two $\lambda/4$ transformers with load 1 in parallel

$$Z_{in1} = \frac{1}{2}\left(\sqrt{2}\right)^2 = 1$$



The Wilkinson power divider

- S parameters

$$Z_{in1} = \frac{1}{2}\left(\sqrt{2}\right)^2 = 1 \qquad S_{11} = 0$$

$$Z_{in2}^{e} = 1 \quad Z_{in2}^{o} = 1$$ and $$Z_{in3}^{e} = 1 \quad Z_{in3}^{o} = 1 \qquad S_{22} = S_{33} = 0$$

$$S_{12} = S_{21} = \frac{V_1^e + V_1^o}{V_2^e + V_2^o} = -\frac{j}{\sqrt{2}}$$

and $$S_{13} = S_{31} = -\frac{j}{\sqrt{2}}$$

$$S_{23} = S_{32} = 0$$ due to short or open at bisection, both eliminate transfer between the ports + reciprocal circuit

The Wilkinson power divider

- at design frequency (length of the transformer equal to $\lambda_o/4$) we have **isolation** between the two output ports

$$[S]=\begin{bmatrix} 0 & -\frac{j}{\sqrt{2}} & -\frac{j}{\sqrt{2}} \\ -\frac{j}{\sqrt{2}} & 0 & 0 \\ -\frac{j}{\sqrt{2}} & 0 & 0 \end{bmatrix}$$

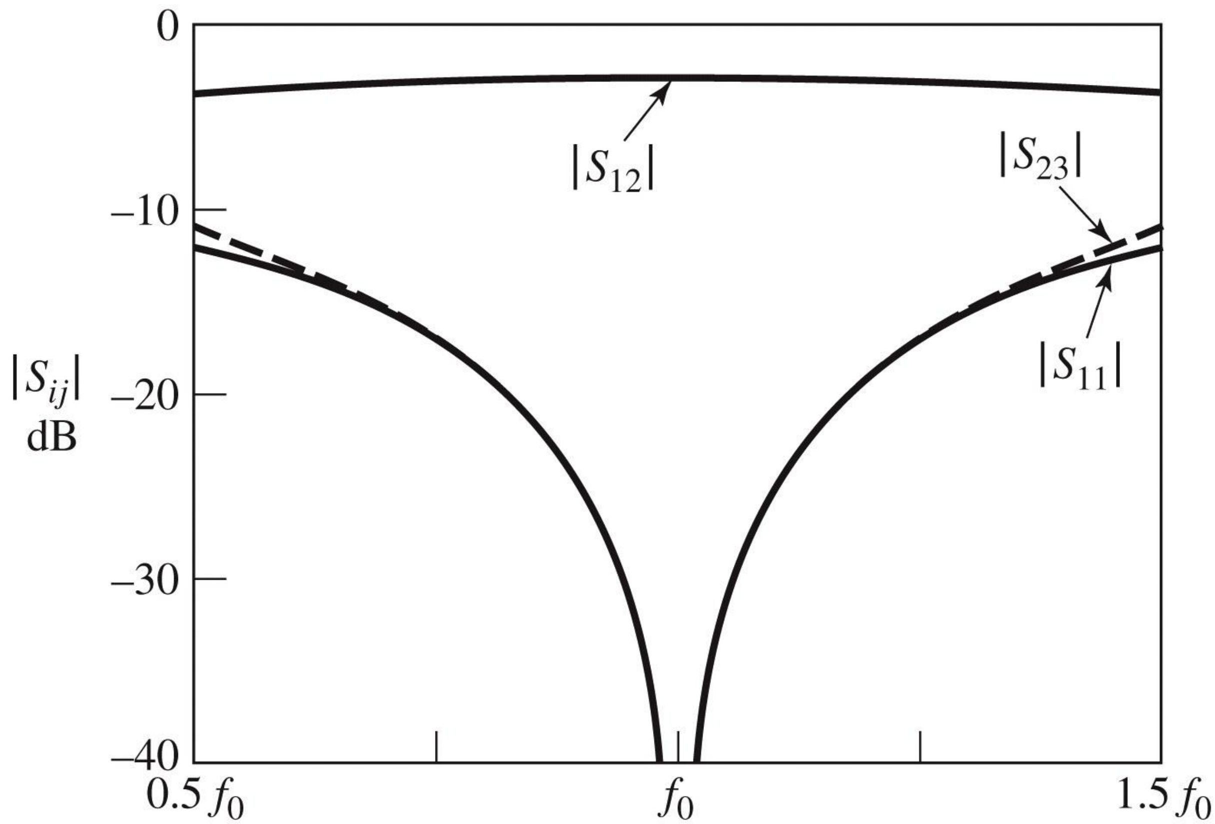


Figure 7.12

The Wilkinson power divider

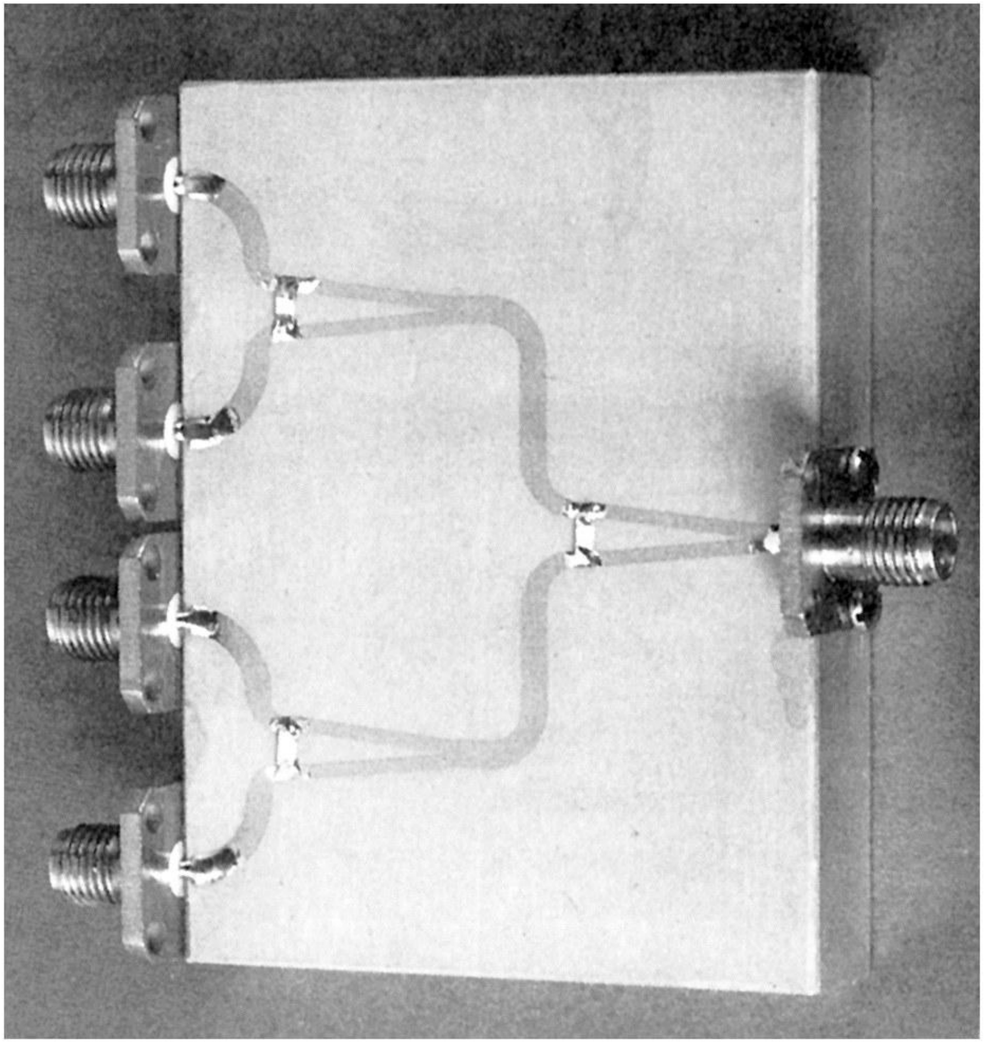
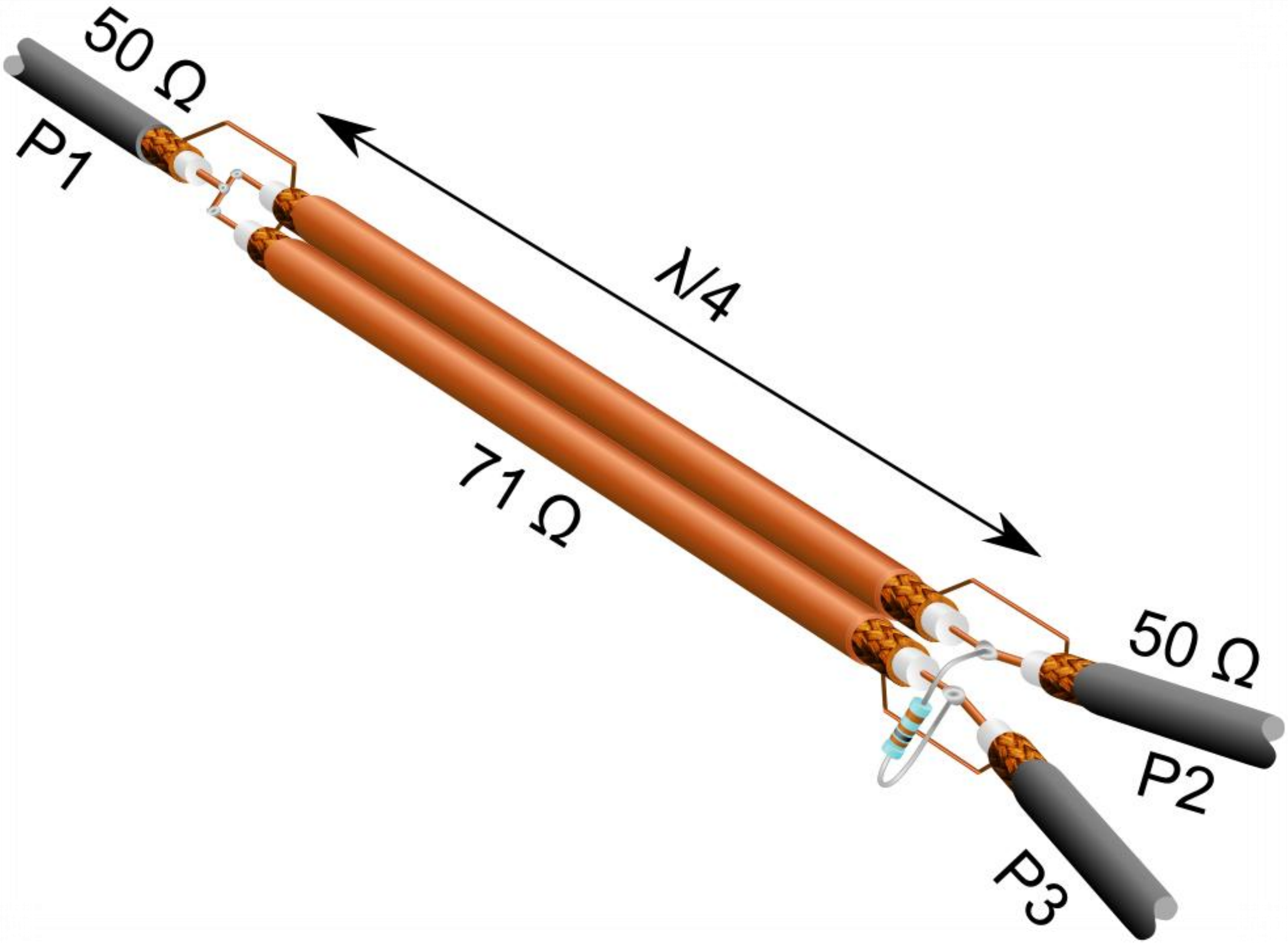


Figure 7.15
Courtesy of M. D. Abouzahra, MIT Lincoln Laboratory, Lexington, Mass.

- 3 X Wilkinson = 4-way power divider

The Wilkinson power divider



Directional couplers

Four-Port Networks

- A four-port network simultaneously:
  - matched at all ports
  - reciprocal
  - lossless
- is **always directional**
  - the signal power injected into one port is transmitted **only towards two** of the other three ports

$$[S] = \begin{bmatrix} 0 & \alpha & \beta \cdot e^{j\theta} & 0 \\ \alpha & 0 & 0 & \beta \cdot e^{j\phi} \\ \beta \cdot e^{j\theta} & 0 & 0 & \alpha \\ 0 & \beta \cdot e^{j\phi} & \alpha & 0 \end{bmatrix}$$

Directional Coupler

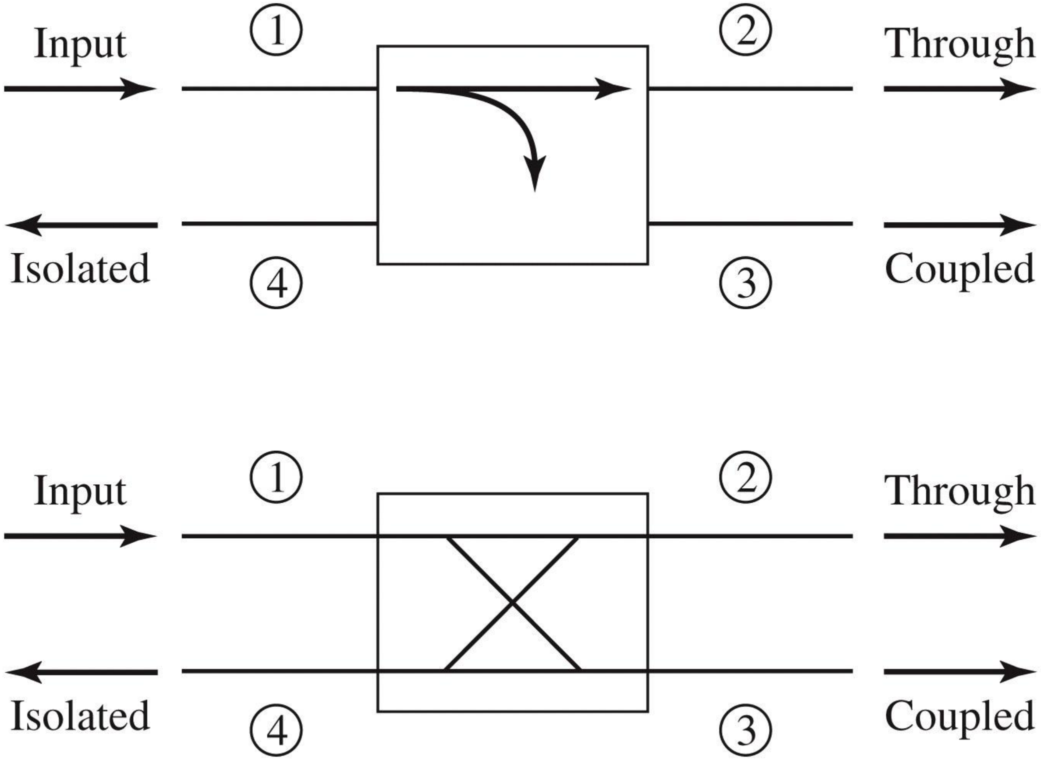


Figure 7.4

$$|S_{12}|^2 = \alpha^2 = 1-\beta^2$$

$$|S_{13}|^2 = \beta^2$$

Coupling

$$C = 10\log\frac{P_1}{P_3} = -20\cdot\log(\beta)\,[\mathrm{dB}]$$

Directivity

$$D = 10\log\frac{P_3}{P_4} = 20\cdot\log\left(\frac{\beta}{|S_{14}|}\right)[\mathrm{dB}]$$

Isolation

$$I = 10\log\frac{P_1}{P_4} = -20\cdot\log|S_{14}|\,[\mathrm{dB}]$$

$$I = D + C\,,\ \ [dB]$$

Four-Port Networks

- two particular choices commonly occur in practice
  - A Symmetric Coupler $\theta = \phi = \pi/2$

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

  - An Antisymmetric Coupler $\theta = 0, \phi = \pi$

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

Hybrid Couplers

Hybrid Couplers are directional couplers with 3 dB coupling factor

$$\alpha = \beta = 1/\sqrt{2}$$

The cuadrature (90°) hybrid

$$\left(\theta = \phi = \pi/2\right)$$

$$[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

The 180° ring hybrid (rat-race)

$$\left(\theta = 0\,, \phi = \pi\right)$$

$$[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

The cuadrature (90°) hybrid

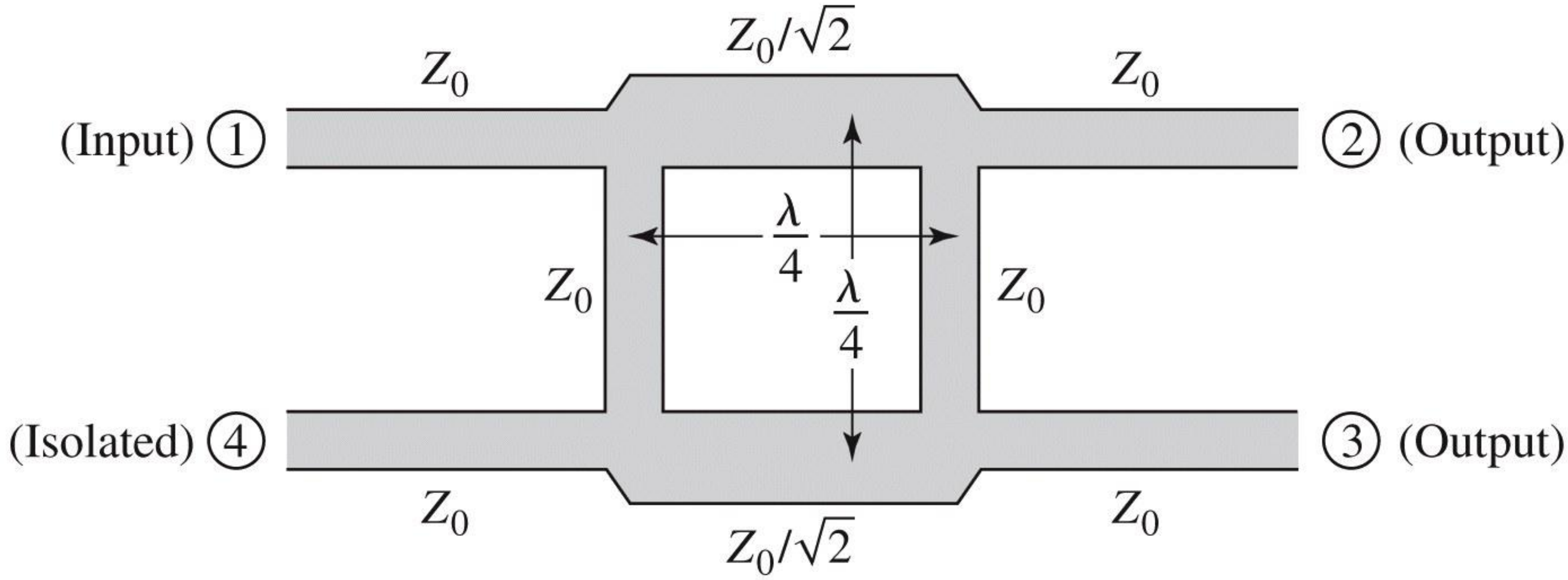
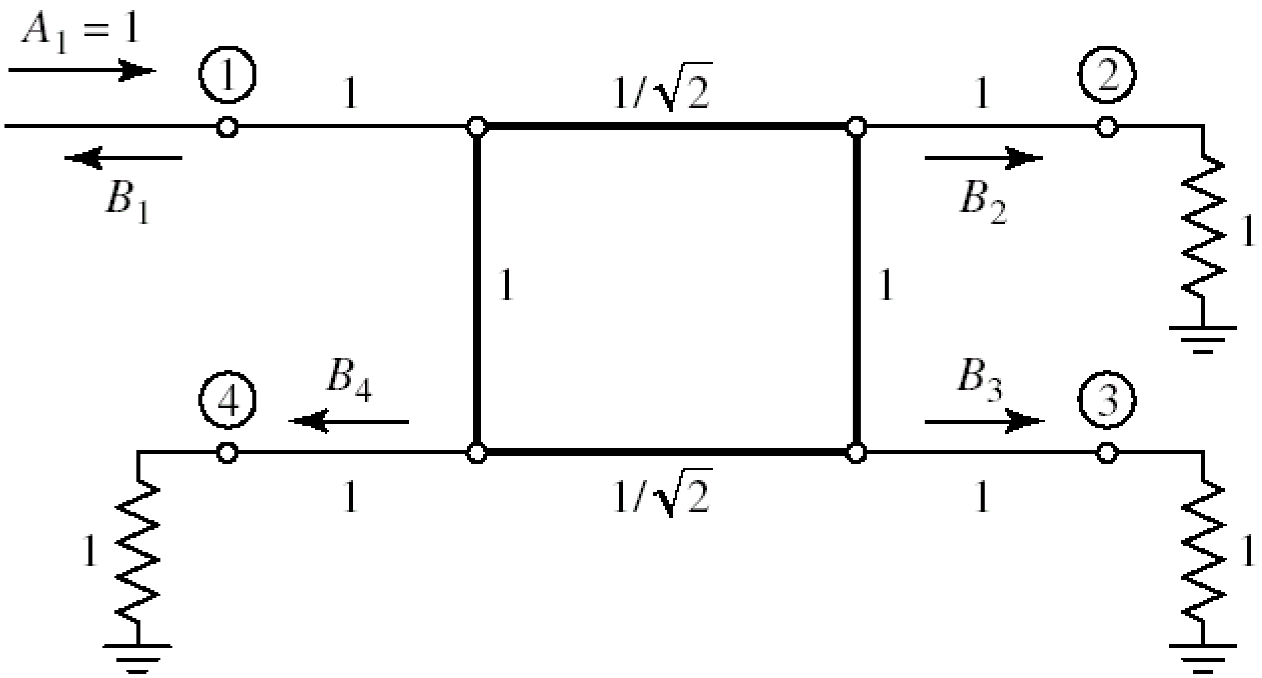


Figure 7.21

$$[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

Even/Odd Mode Analysis



Even/Odd Mode Analysis

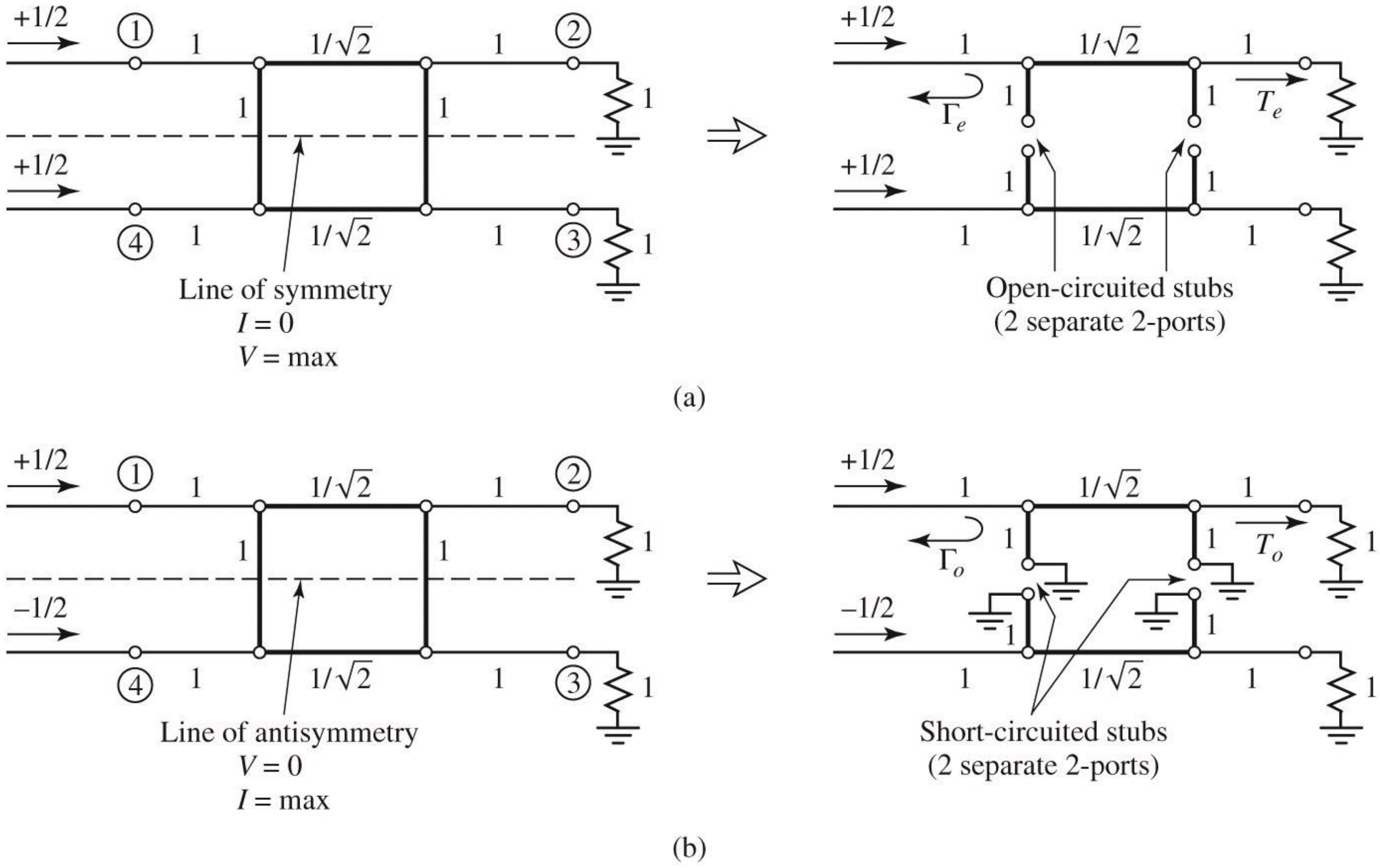
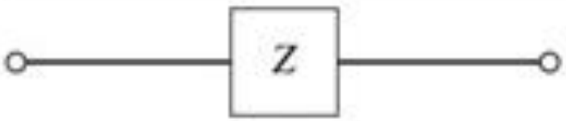
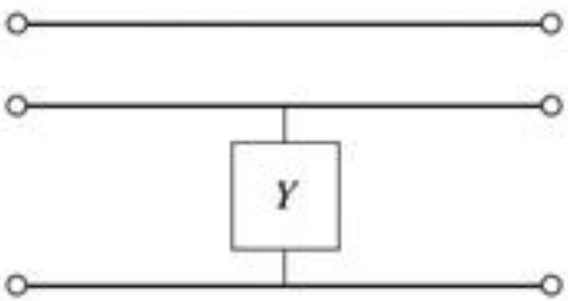
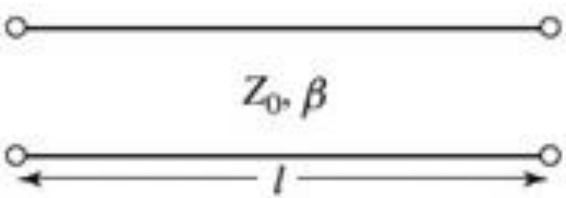


Figure 7.23

$$b_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o \qquad b_2 = \frac{1}{2}T_e + \frac{1}{2}T_o \qquad b_3 = \frac{1}{2}T_e - \frac{1}{2}T_o \qquad b_4 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o$$

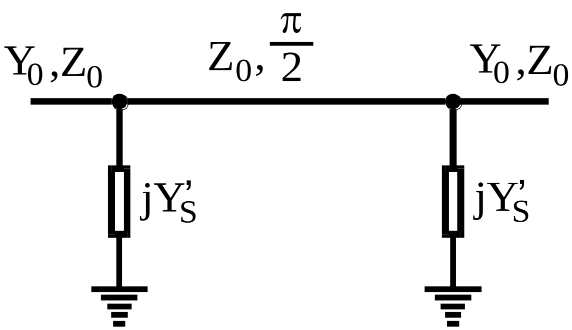
Library of ABCD matrices

TABLE 4.1 ***ABCD* Parameters of Some Useful Two-Port Circuits**

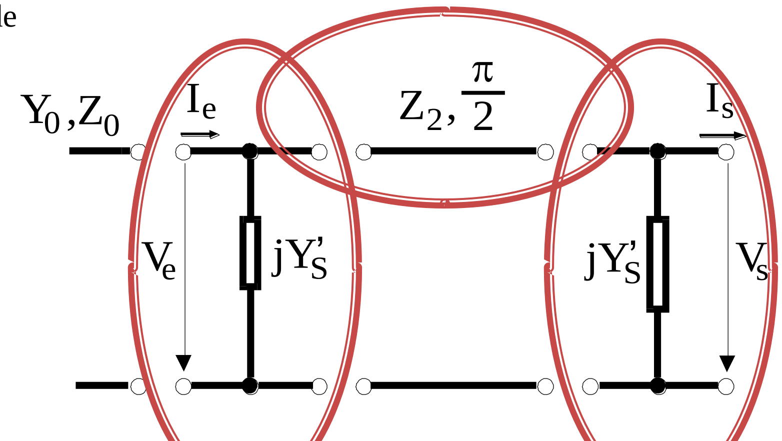
Circuit	*ABCD* Parameters	
	$A = 1$ $C = 0$	$B = Z$ $D = 1$
	$A = 1$ $C = Y$	$B = 0$ $D = 1$
	$A = \cos \beta\ell$ $C = jY_0 \sin \beta\ell$	$B = jZ_0 \sin \beta\ell$ $D = \cos \beta\ell$

S parameters (from ABCD)

$$Y'_S = \begin{cases} Y_1 & \text{even mode} \\ -Y_1 & \text{odd mode} \end{cases}$$



a)



b)

$$\begin{bmatrix} V_e \\ I_e \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ jY'_S & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & jZ_2 \\ jY_2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ jY'_S & 1 \end{bmatrix} \cdot \begin{bmatrix} V_S \\ I_S \end{bmatrix}$$

$$\begin{bmatrix} V_e \\ I_e \end{bmatrix} = \begin{bmatrix} -Y'_S Z_2 & jZ_2 \\ -jY'^2_S Z_2 + jY_2 & -Y'_S Z_2 \end{bmatrix} \cdot \begin{bmatrix} V_S \\ I_S \end{bmatrix}$$

$$S_{11} = \frac{j\dfrac{Z_2}{Z_0} - Z_0\left(-jY'^2_S Z_2 + jY_2\right)}{-2Y'_S Z_2 + j\dfrac{Z_2}{Z_0} + Z_0\left(-jY'^2_S + jY_2\right)}$$

$$S_{12} = \frac{2\left[\left(-Y'_S Z_2\right)^2 - jZ_2\left(-jY'^2_S Z_2 + jY_2\right)\right]}{-2Y'_S Z_2 + j\dfrac{Z_2}{Z_0} + Z_0\left(-jY'^2_S Z_2 + jY_2\right)}$$

$$\Gamma = S_{11} = \frac{j\left(z_2 - y_2 + y'^2_S z_2\right)}{-2y'_S z_2 + j\left(z_2 + y_2 - y'^2_S z_2\right)} = S_{22}$$

$$S_{21} = \frac{2}{-2Y'_S Z_2 + j\dfrac{Z_2}{Z_0} + Z_0\left(-jY'^2_S Z_2 + jY_2\right)}$$

$$S_{22} = \frac{j\dfrac{Z_2}{Z_0} - Z_0\left(-jY'^2_S Z_2 + jY_2\right)}{-2Y'_S Z_2 + j\dfrac{Z_2}{Z_0} + Z_0\left(-jY'^2_S Z_2 + jY_2\right)}$$

$$T = S_{21} = \frac{2}{-2y'_S z_2 + j\left(z_2 + y_2 - y'^2_S z_2\right)} = S_{12}$$

Relation between two port S parameters and ABCD parameters

$$A = \sqrt{\frac{Z_{01}}{Z_{02}}}\,\frac{(1 + S_{11} - S_{22} - \Delta S)}{2S_{21}}$$

$$B = \sqrt{Z_{01}Z_{02}}\,\frac{(1 + S_{11} + S_{22} + \Delta S)}{2S_{21}}$$

$$C = \frac{1}{\sqrt{Z_{01}Z_{02}}}\,\frac{1 - S_{11} - S_{22} + \Delta S}{2S_{21}}$$

$$D = \sqrt{\frac{Z_{02}}{Z_{01}}}\,\frac{1 - S_{11} + S_{22} - \Delta S}{2S_{21}}$$

$$\Delta S = S_{11}S_{22} - S_{12}S_{21}$$

$$S_{11} = \frac{AZ_{02} + B - CZ_{01}Z_{02} - DZ_{01}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}}$$

$$S_{12} = \frac{2(AD - BC)\sqrt{Z_{01}Z_{02}}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}}$$

$$S_{21} = \frac{2\sqrt{Z_{01}Z_{02}}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}}$$

$$S_{22} = \frac{-AZ_{02} + B - CZ_{01}Z_{02} + DZ_{01}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}}$$

Matching and coupling factor

$$\Gamma_e = \frac{j\cdot\left(z_2 - y_2 + y_1^2 z_2\right)}{-2y_1z_2 + j\left(z_2 + y_2 - y_1^2 z_2\right)}$$

$$\Gamma_o = \frac{j\cdot\left(z_2 - y_2 + y_1^2 z_2\right)}{2y_1z_2 + j\left(z_2 + y_2 - y_1^2 z_2\right)}$$

$$T_e = \frac{2}{-2y_1z_2 + j\cdot\left(z_2 + y_2 - y_1^2 z_2\right)}$$

$$T_o = \frac{2}{2y_1z_2 + j\cdot\left(z_2 + y_2 - y_1^2 z_2\right)}$$

$$b_1 = \frac{\Gamma_e + \Gamma_o}{2} = \frac{z_2^2 - \left(y_2 - y_1^2 z_2\right)^2}{(2y_1z_2)^2 + \left(z_2 + y_2 - y_1^2 z_2\right)^2}$$

$$b_2 = \frac{T_e + T_o}{2} = \frac{-2j\left(z_2 + y_2 - y_1^2 z_2\right)}{(2y_1z_2)^2 + \left(z_2 + y_2 - y_1^2 z_2\right)^2}$$

$$b_3 = \frac{T_e - T_o}{2} = \frac{-4y_1z_2}{(2y_1z_2)^2 + \left(z_2 + y_2 - y_1^2 z_2\right)^2}$$

$$b_4 = \frac{\Gamma_e - \Gamma_o}{2} = \frac{-2jy_1z_2\left(z_2 - y_2 + y_1^2 z_2\right)}{(2y_1z_2)^2 + \left(z_2 + y_2 - y_1^2 z_2\right)^2}$$

$$b_1 = 0 \Rightarrow z_2 - y_2 + y_1^2 z_2 = 0 \Rightarrow z_2^2 = \frac{1}{1 + y_1^2}$$

$$\boxed{y_2^2 = 1 + y_1^2}$$

$$\boxed{C = 10\log\frac{P_1}{P_3} = -20\log|b_3|\ , dB}$$

$$b_1 = 0 \quad b_4 = 0 \quad b_3 = -y_1 z_2 \quad b_2 = -jz_2$$

$$b_3 = -\frac{\sqrt{y_2^2 - 1}}{y_2}\ ,\ b_2 = -\frac{j}{y_2}$$

$$\boxed{\beta = \frac{\sqrt{y_2^2 - 1}}{y_2}}$$

$$b_3 = -C$$

$$b_2 = -j\sqrt{1 - C^2}$$

$$[S] = \begin{bmatrix} 0 & -j\sqrt{1-C^2} & -C & 0 \\ -j\sqrt{1-C^2} & 0 & 0 & -C \\ -C & 0 & 0 & -j\sqrt{1-C^2} \\ 0 & -C & -j\sqrt{1-C^2} & 0 \end{bmatrix}$$

The cuadrature (90°) hybrid

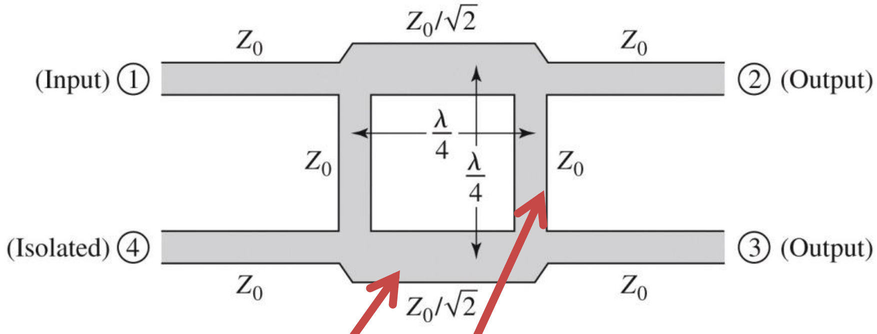


Figure 7.21

$$\boxed{y_2^2 = 1 + y_1^2}$$

$$|\beta| = \frac{\sqrt{y_2^2 - 1}}{y_2}$$

$$\boxed{C[\mathrm{dB}] = -20 \cdot \log_{10} \frac{\sqrt{y_2^2 - 1}}{y_2}}$$

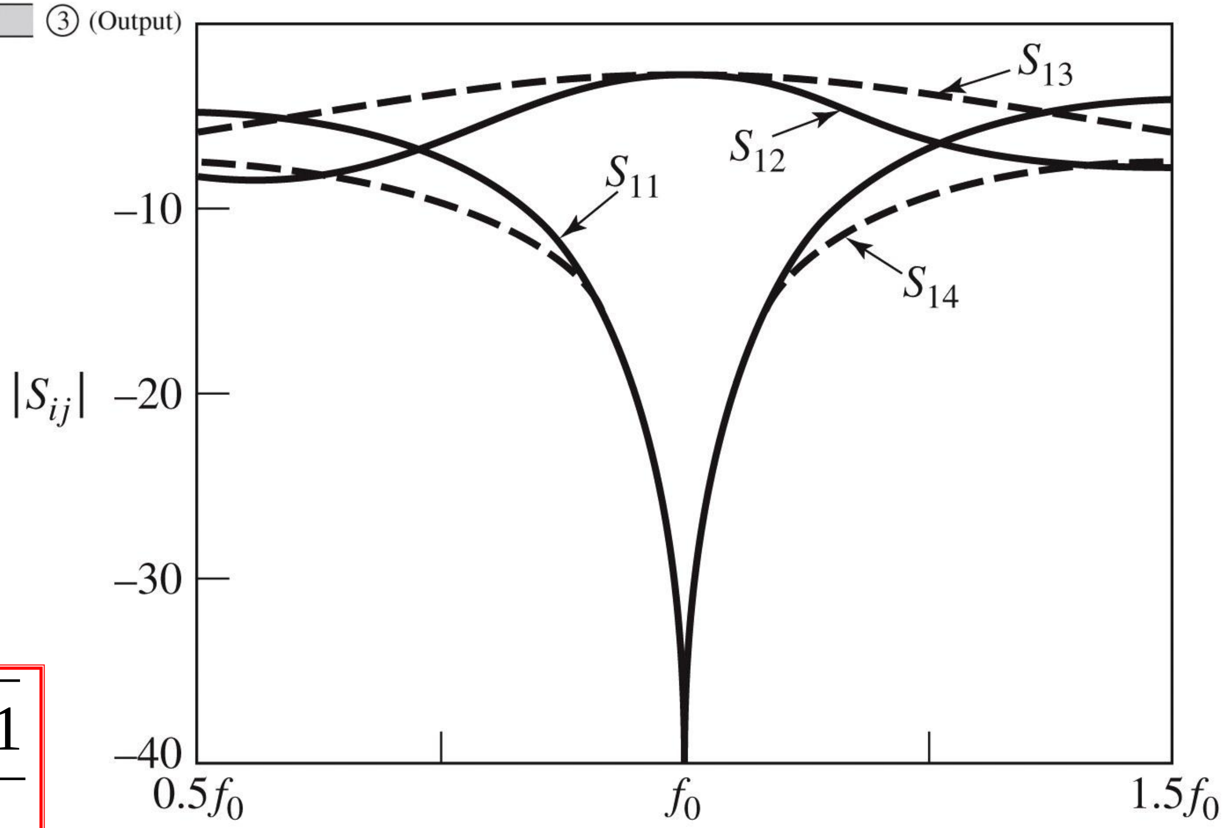


Figure 7.25

Example

Design a cuadrature (90°) hybrid working on 50 Ω, and plot the S parameters between

$0.5\,f_0$ and $1.5\,f_0$, where f_0

is the frequency at which the length of the branches is $\lambda/4$

.

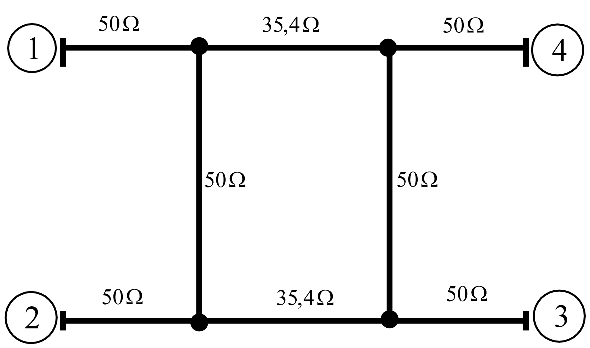
Solution

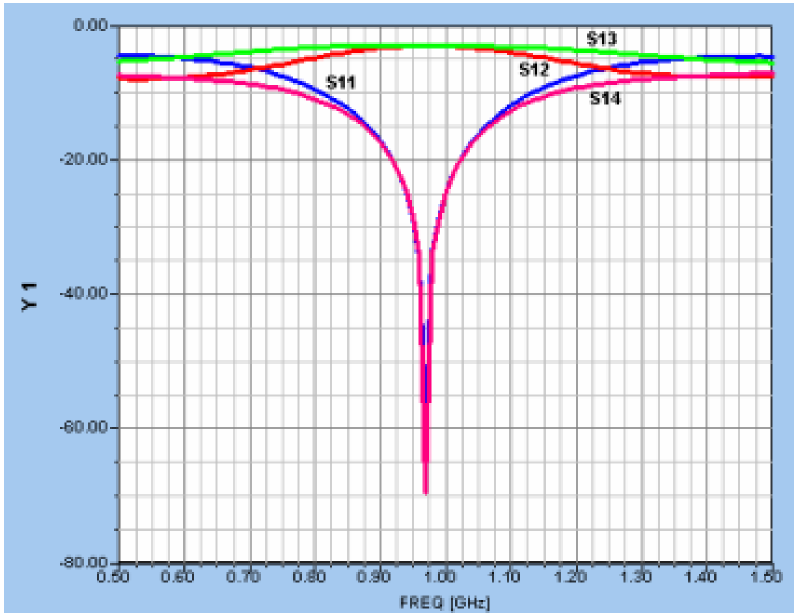
A cuadrature (90°) hybrid has C = 3dB, then $\beta = 1/\sqrt{2}$

$y_2 = \sqrt{2}$ and $y_1 = 1$

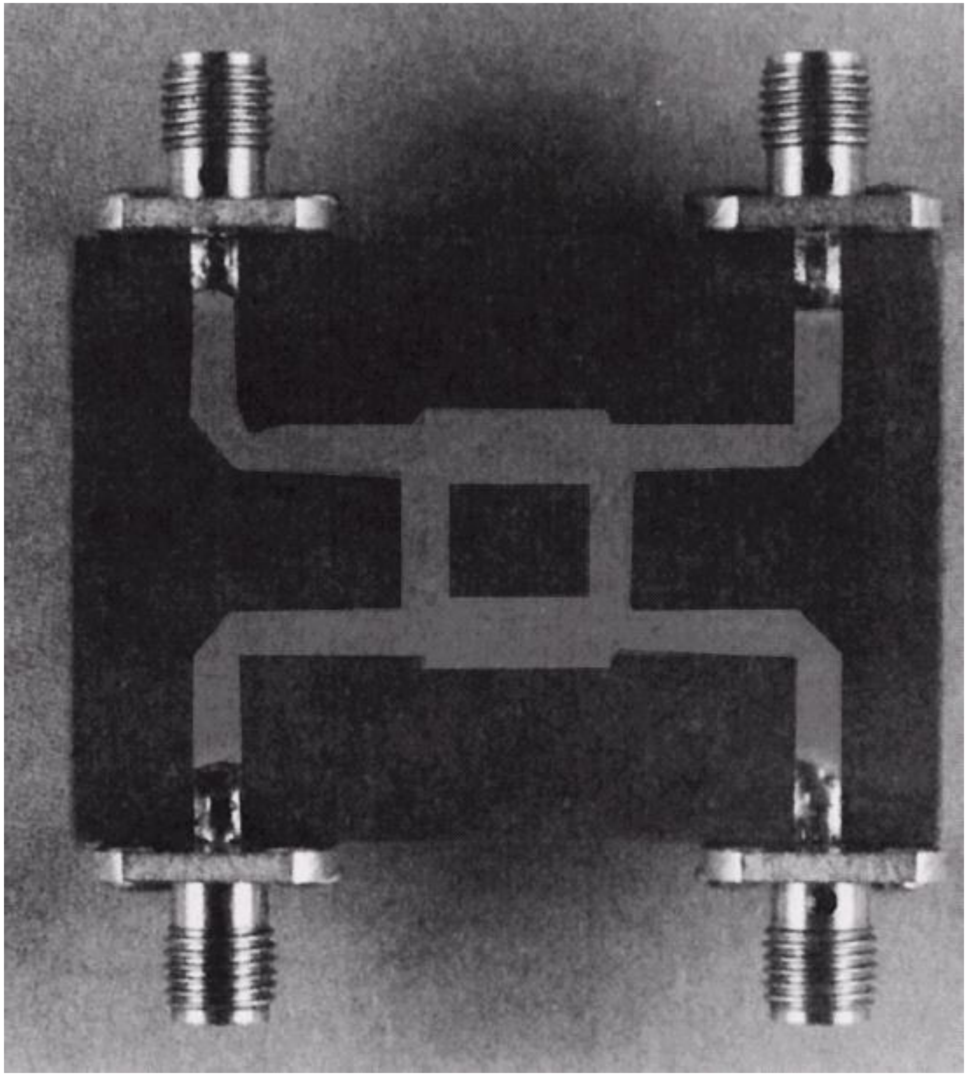
$Z_0 = 50\Omega$ the characteristic impedances will be:

$Z_1 = Z_0 = 50\Omega$ $\quad Z_2 = \dfrac{Z_0}{\sqrt{2}} = 35.4\Omega$





The cuadrature (90°) hybrid



The cuadrature (90°) hybrid

- eight-way microstrip power divider with six quadrature hybrids in a Bailey configuration

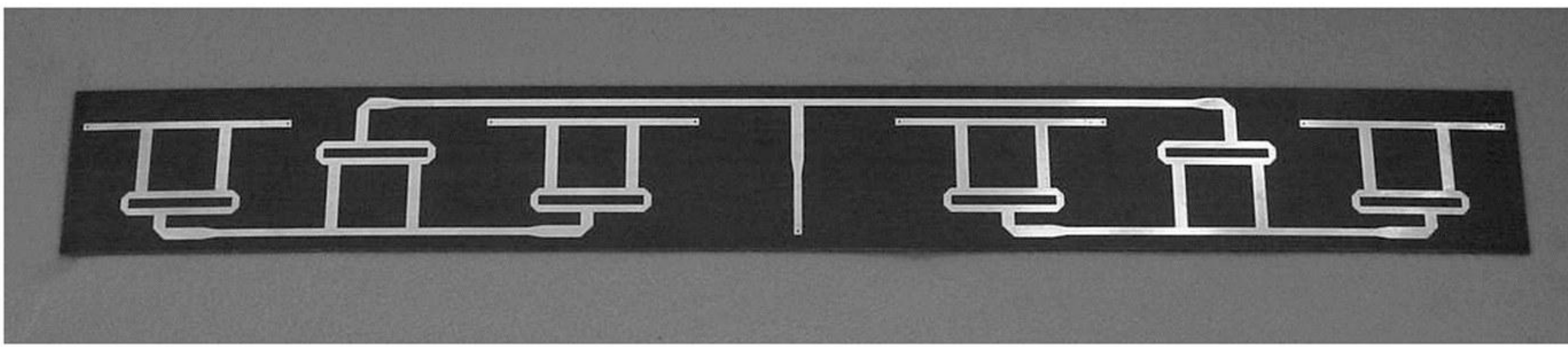
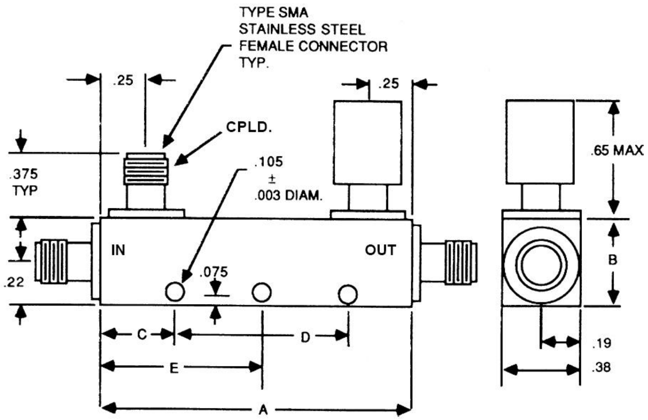


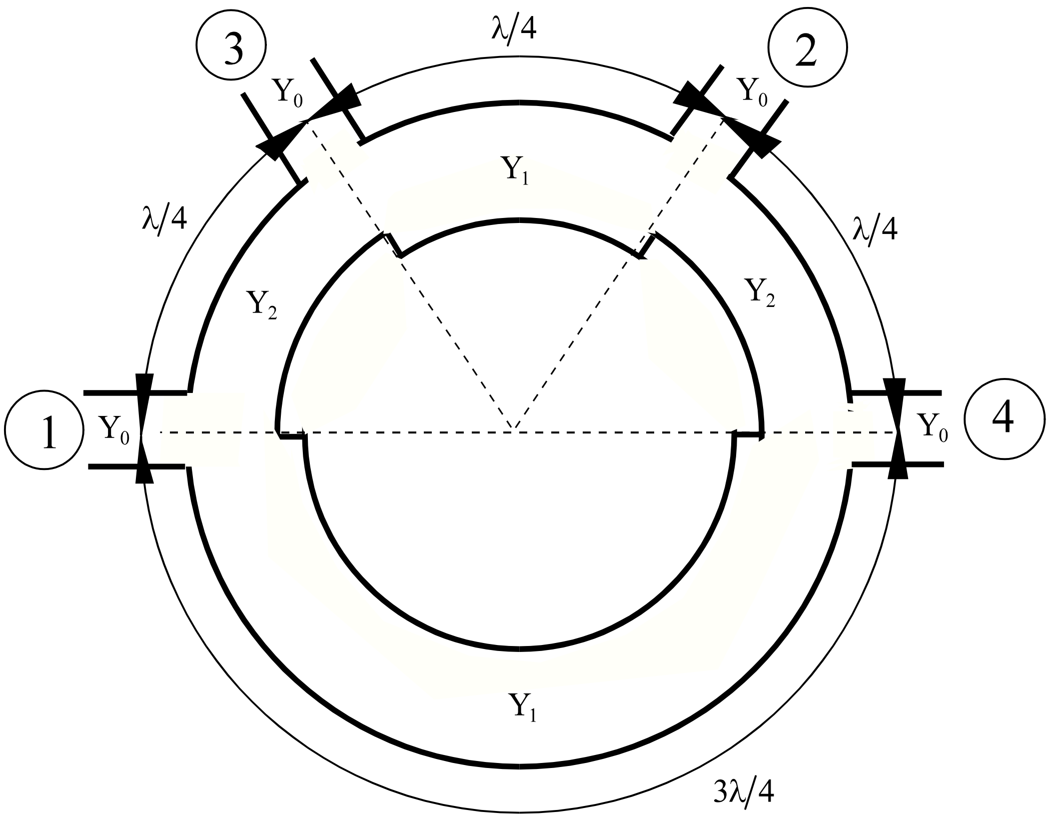
Figure 7.24
Courtesy of ProSensing, Inc., Amherst, Mass.

Datasheet



Model No.	Frequency Range (Ghz)	Coupling † (dB)	Freq. Sens. (dB)	Insertion Loss (dB)		Directivity (dB min.)	VSWR max.	
				Excl. Cpld Pwr	True		Primary Line	Secondary Line
MDC6223-6	0.5-1.0	6 ±1.00	±0.60	0.20	1.80	25	1.15	1.15
MDC6223-10	0.5-1.0	10 ±1.25	±0.75	0.20	0.80	25	1.10	1.10
MDC6223-20	0.5-1.0	20 ±1.25	±0.75	0.15	0.20	25	1.10	1.10
MDC6223-30	0.5-1.0	30 ±1.25	±0.75	0.15	0.20	25	1.10	1.10
MDC6224-6	1.0-2.0	6 ±1.00	±0.60	0.20	1.80	25	1.15	1.15
MDC6224-10	1.0-2.0	10 ±1.25	±0.75	0.20	0.80	25	1.10	1.10
MDC6224-20	1.0-2.0	20 ±1.25	±0.75	0.15	0.20	25	1.10	1.10
MDC6224-30	1.0-2.0	30 ±1.25	±0.75	0.15	0.20	25	1.10	1.10
MDC6225-6	2.0-4.0	6 ±1.00	±0.60	0.20	1.80	22	1.15	1.15
MDC6225-10	2.0-4.0	10 ±1.25	±0.75	0.20	0.80	22	1.15	1.15
MDC6225-20	2.0-4.0	20 ±1.25	±0.75	0.15	0.20	22	1.15	1.15
MDC6225-30	2.0-4.0	30 ±1.25	±0.75	0.15	0.20	22	1.15	1.15
MDC6266-6	2.6-5.2	6 ±1.00	±0.60	0.20	1.80	20	1.25	1.25
MDC6266-10	2.6-5.2	10 ±1.25	±0.75	0.20	0.80	20	1.25	1.25
MDC6266-20	2.6-5.2	20 ±1.25	±0.75	0.20	0.25	20	1.25	1.25
MDC6266-30	2.6-5.2	30 ±1.25	±0.75	0.20	0.20	20	1.25	1.25
MDC6226-6	4.0-8.0	6 ±1.00	±0.60	0.25	1.90	20	1.25	1.25
MDC6226-10	4.0-8.0	10 ±1.25	±0.75	0.25	0.90	20	1.25	1.25
MDC6226-20	4.0-8.0	20 ±1.25	±0.75	0.25	0.30	20	1.25	1.25
MDC6226-30	4.0-8.0	30 ±1.25	±0.75	0.25	0.25	20	1.25	1.25
MDC6227-6	7.0-12.4	6 ±1.00	±0.50	0.30	2.00	17	1.30	1.30
MDC6227-10	7.0-12.4	10 ±1.00	±0.50	0.30	1.00	17	1.30	1.30
MDC6227-20	7.0-12.4	20 ±1.00	±0.50	0.30	0.35	17	1.30	1.30

The 180° ring hybrid (rat-race)



The 180° ring hybrid

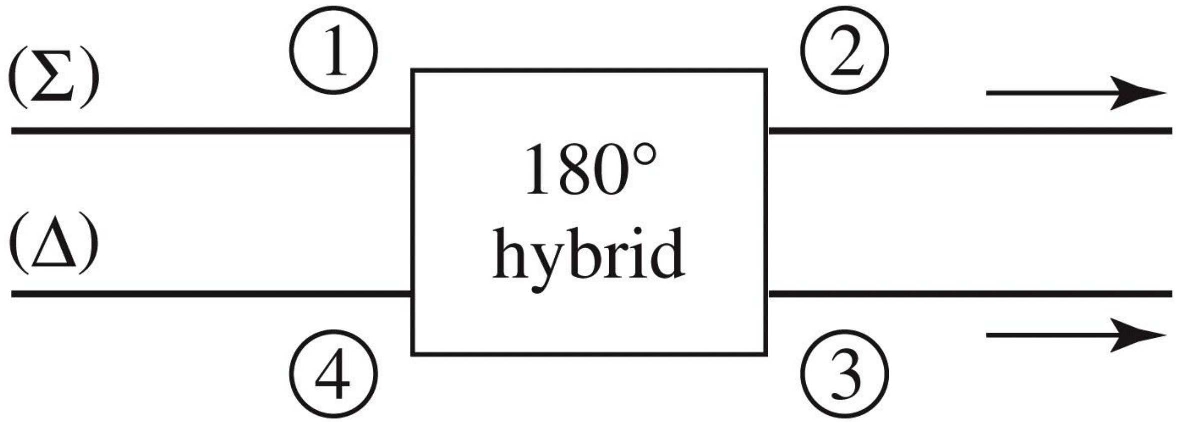
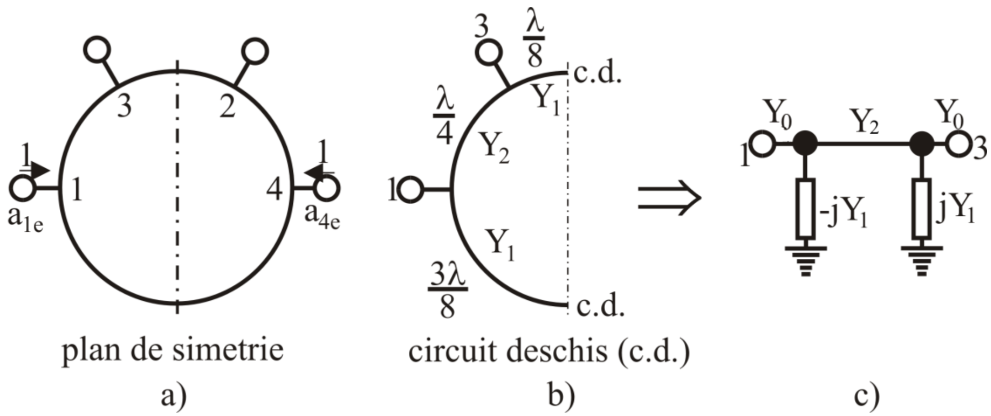


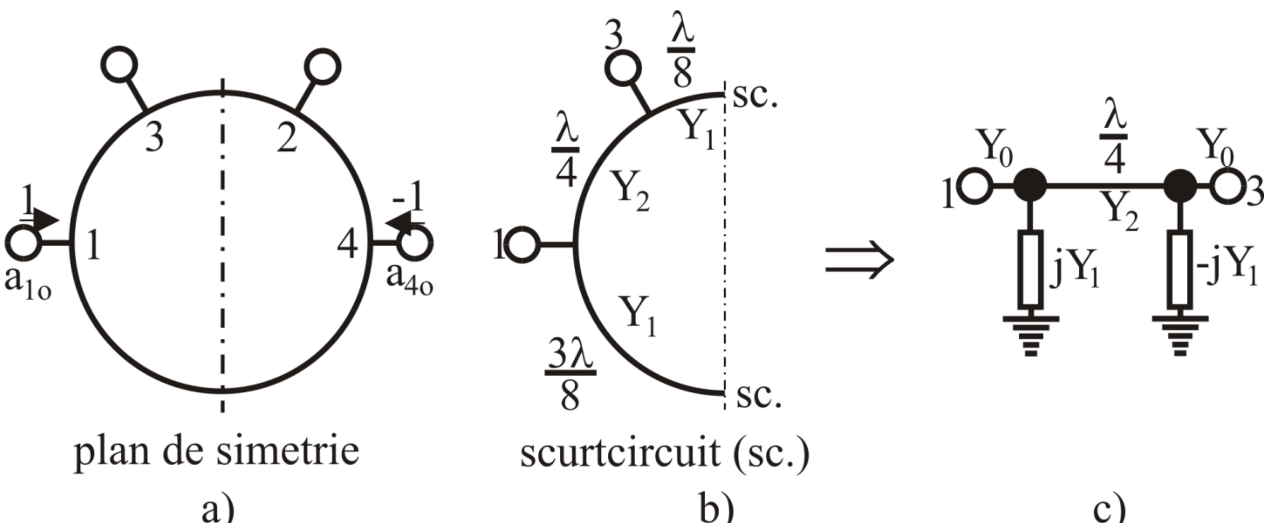
Figure 7.41

- The 180° ring hybrid can be operated in different modes:
  - a signal applied to port 1 will be evenly split into two in-phase components at ports 2 and 3
  - input applied to port 4 it will be equally split into two components with a 180◦ phase difference at ports 2 and 3
  - input signals applied at ports 2 and 3, the sum of the inputs will be formed at port 1, while the difference will be formed at port 4 (power combiner)

Even/Odd Mode Analysis



Even Mode

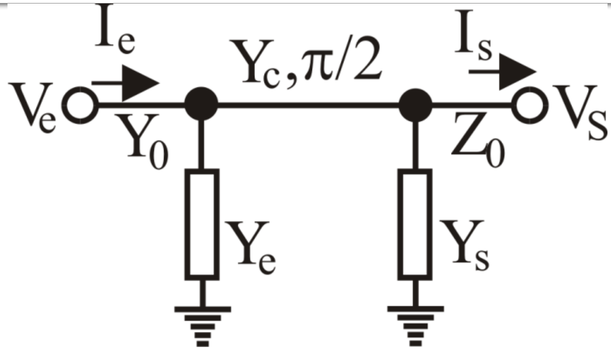


Odd Mode

Even/Odd Mode Analysis

$$S_{11} = \frac{jz_2 y_s + jz_2 - j(y_2 + y_e y_s z_2) - jy_e z_2}{jz_2 y_s + jz_2 + j(y_2 + y_e y_s z_2) + jy_e z_2}$$

$$S_{12} = \frac{2}{jz_2 y_s + jz_2 + j(y_2 + y_e y_s z_2) + jy_e z_2}$$



$$S_{21} = \frac{2}{jz_2 y_s + jz_2 + j(y_2 + y_e y_s z_2) + jy_e z_2}$$

$$S_{22} = \frac{-jz_2 y_s + jz_2 - j(y_2 + y_e y_s z_2) + jy_e z_2}{jz_2 y_s + jz_2 + j(y_2 + y_e y_s z_2) + jy_e z_2}$$

Even mode:

$$y_e = -jy_1$$

$$y_s = jy_1$$

$$S_{11e} = \frac{z_2 - y_2 - y_1^2 z_2 + 2jz_2 y_1}{z_2 + y_2 + y_1^2 z_2}$$

$$S_{12e} = S_{21e} = \frac{-2j}{z_2 + y_2 + y_1^2 z_2}$$

$$S_{22e} = \frac{z_2 - y_2 - y_1^2 z_2 - 2jz_2 y_1}{z_2 + y_2 + y_1^2 z_2}$$

Matching condition

$$y_1^2 + y_2^2 = 1$$

$$[S] = \begin{bmatrix} 0 & 0 & -jy_2 & jy_1 \\ 0 & 0 & -jy_1 & -jy_2 \\ -jy_2 & -jy_1 & 0 & 0 \\ jy_1 & -jy_2 & 0 & 0 \end{bmatrix}$$

Odd mode:

$$y_e = jy_1$$

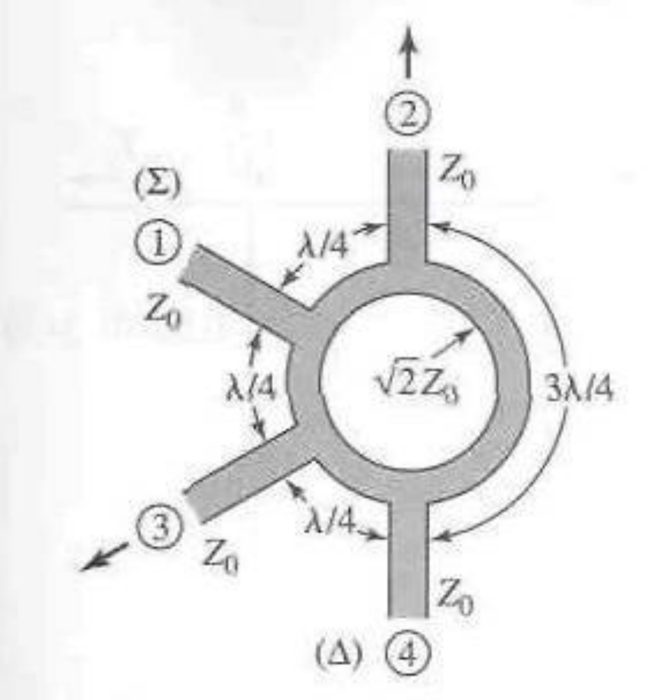
$$y_s = -jy_1$$

$$S_{11o} = \frac{z_2 - y_2 - y_1^2 z_2 - 2jz_2 y_1}{z_2 + y_2 + y_1^2 z_2}$$

$$S_{12o} = S_{21o} = \frac{-2j}{z_2 + y_2 + y_1^2 z_2}$$

$$S_{22o} = \frac{z_2 - y_2 - y_1^2 z_2 + 2jz_2 y_1}{z_2 + y_2 + y_1^2 z_2}$$

The 180° ring hybrid



$$[S]=\begin{bmatrix} 0 & -jy_2 & -jy_1 & 0 \\ -jy_2 & 0 & 0 & jy_1 \\ -jy_1 & 0 & 0 & -jy_2 \\ 0 & jy_1 & -jy_2 & 0 \end{bmatrix} = -j\begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

$$C(dB) = -20\log(\beta) = -20\log(y_1)$$

Example

Design a ring (180°) hybrid working on 50 Ω, and plot the S parameters between 0.5 and 1.5 of the design frequency.

$$C\,[\text{dB}] = -20\log(y_1)$$

$$\sqrt{2}Z_0 = 70.7\Omega$$

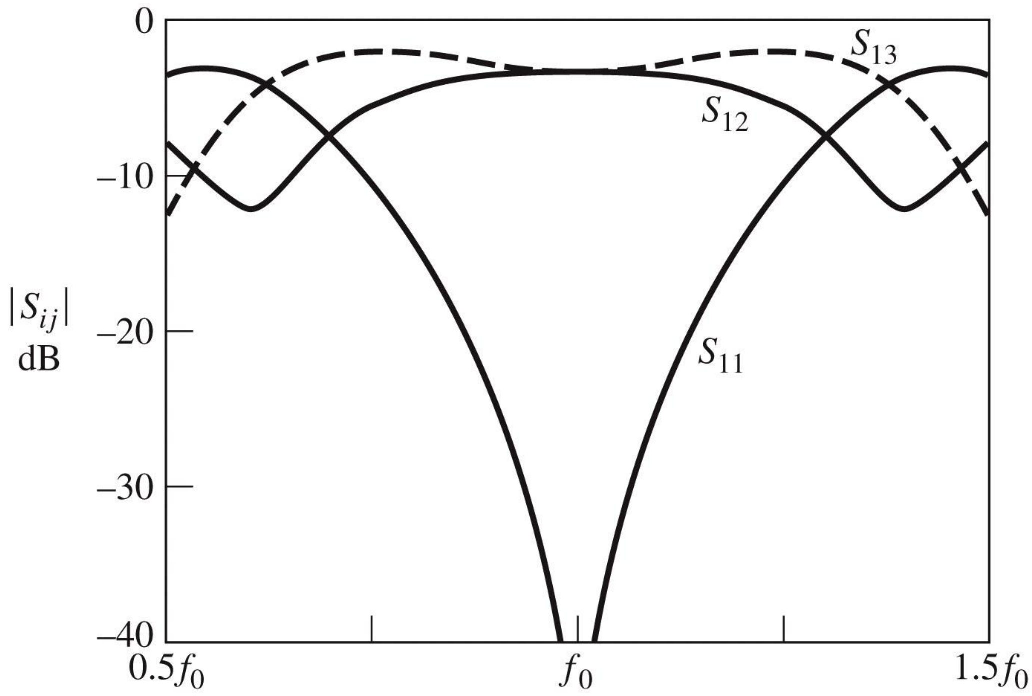
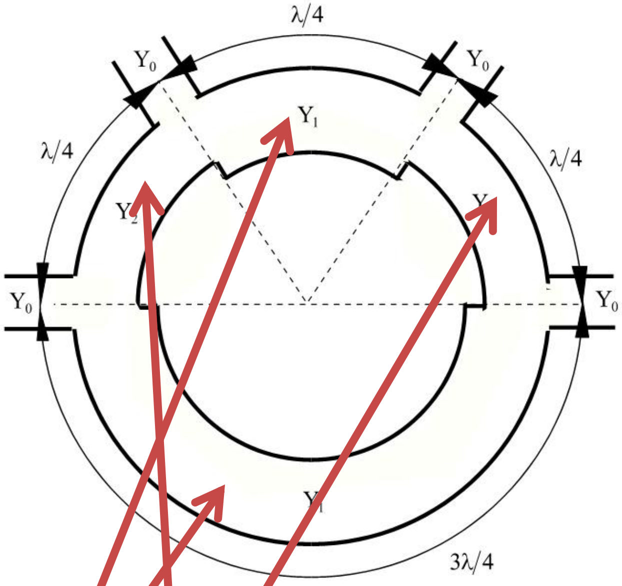


Figure 7.46

The 180° ring hybrid



$$\boxed{y_1^2 + y_2^2 = 1}$$

$$|\beta| = y_1$$

$$\boxed{C\,[\mathrm{dB}] = -20 \cdot \log_{10}(y_1)}$$

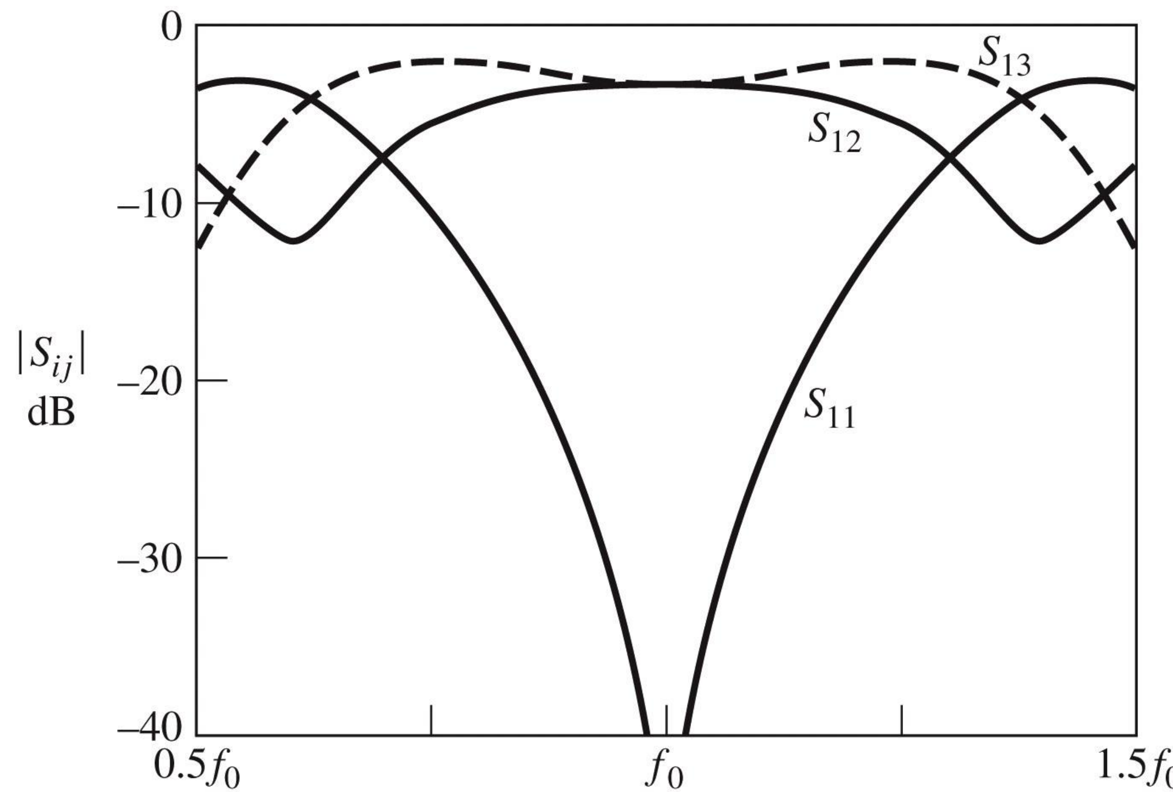


Figure 7.46

The 180° ring hybrid

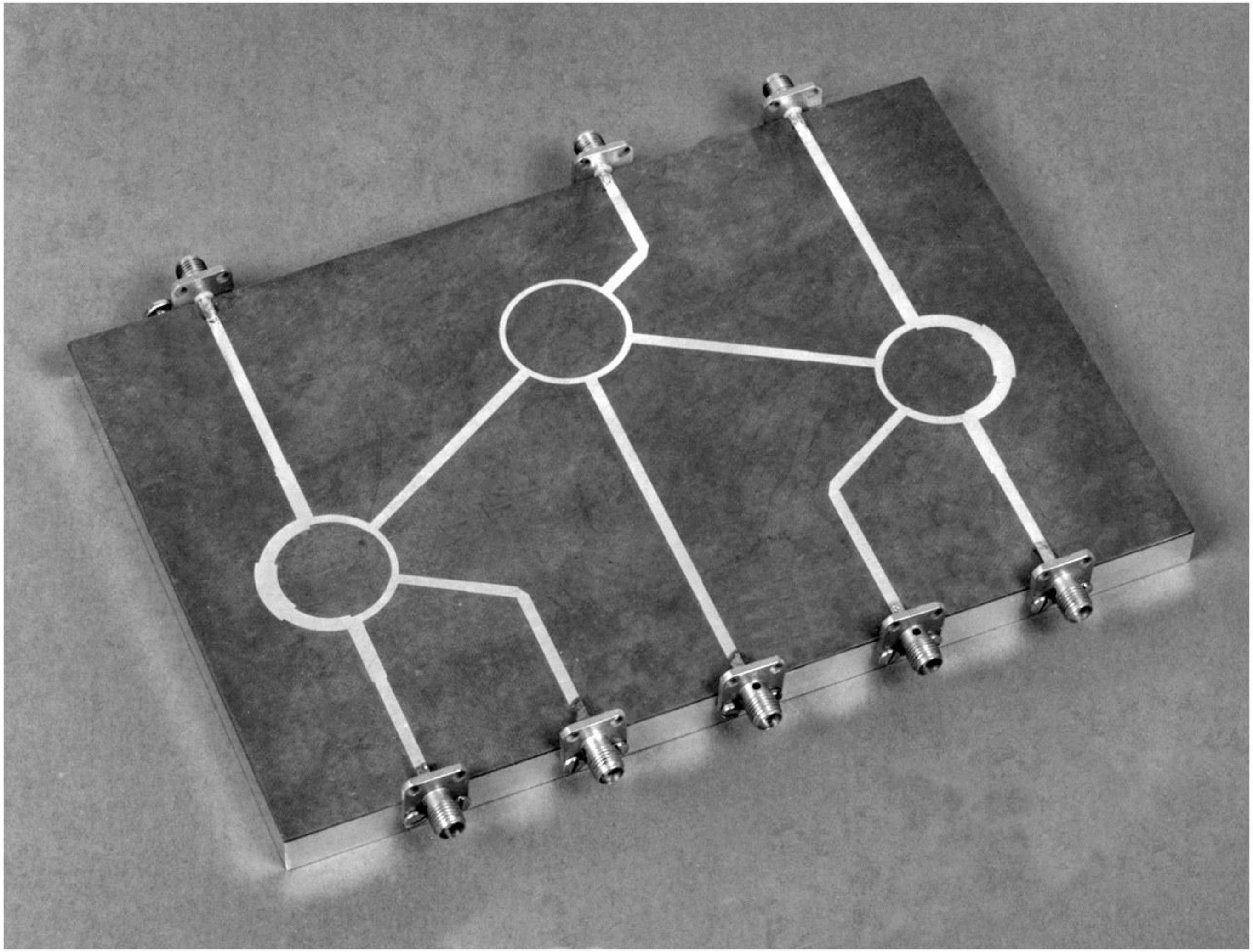
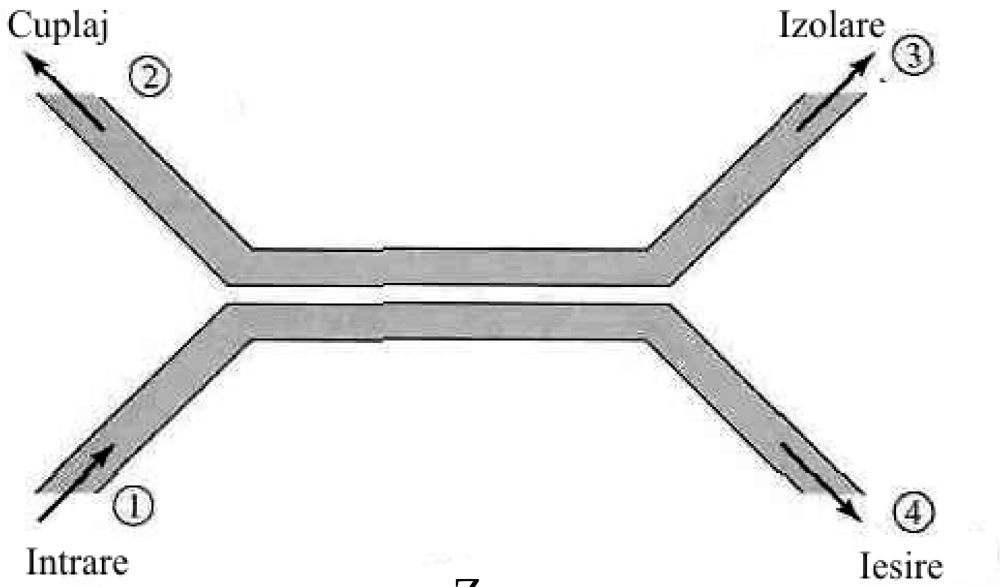
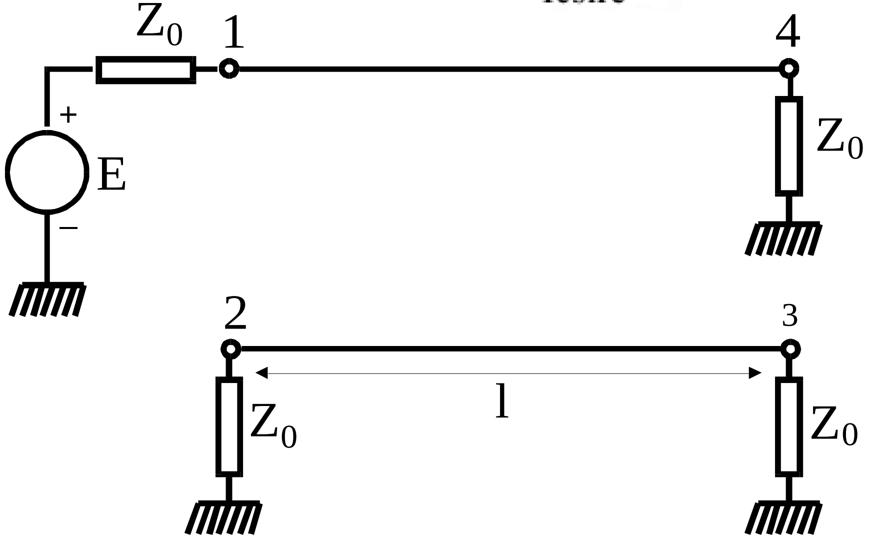


Figure 7.43
Courtesy of M. D. Abouzahra, MIT Lincoln Laboratory, Lexington, Mass.

Coupled Line Coupler





Coupled Lines

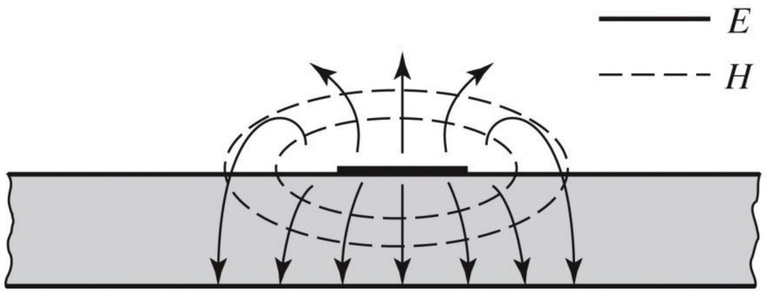
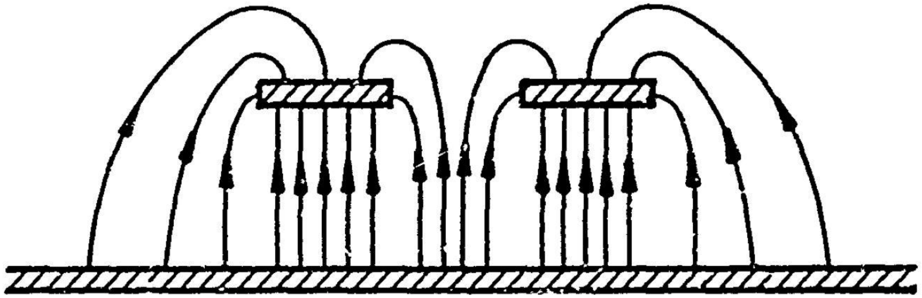
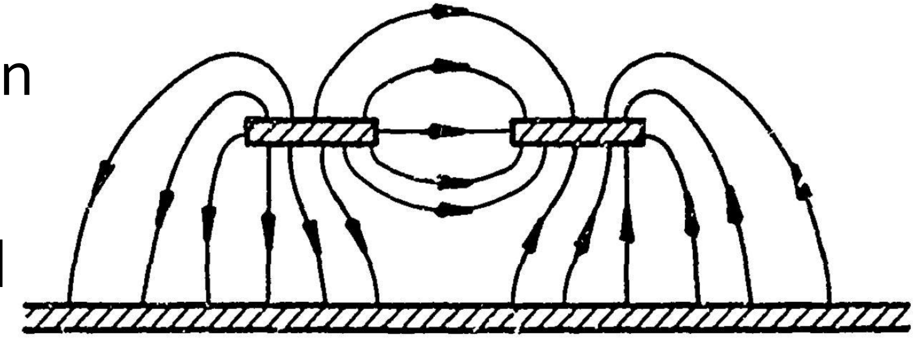


Figure 3.25b



b) EVEN MODE ELECTRIC FIELD PATTERN (SCHEMATIC)



c) ODD MODE ELECTRIC FIELD PATTERN (SCHEMATIC)

- Even mode - characterizes the common mode signal on the two lines
- Odd mode - characterizes the differential mode signal between the two lines
- Each of the two modes is characterized by **different** characteristic impedances

Coupled Lines

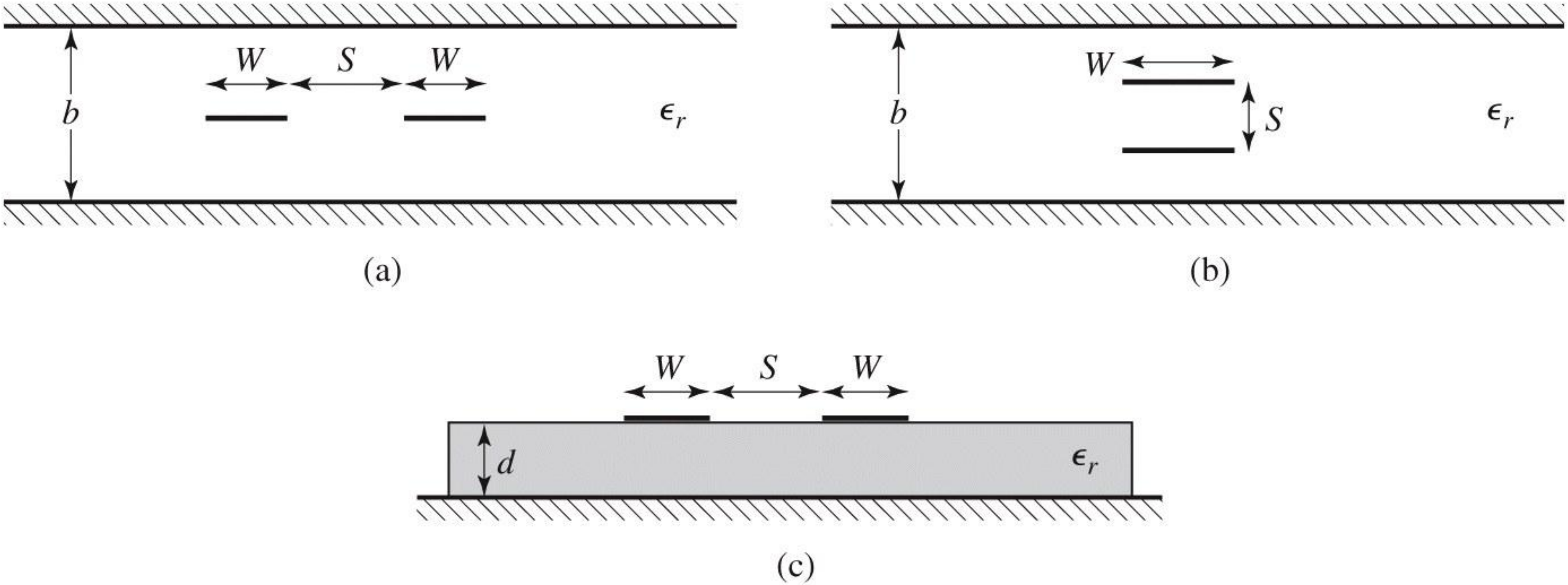


Figure 7.26

Coupled Lines

Figure 7.27

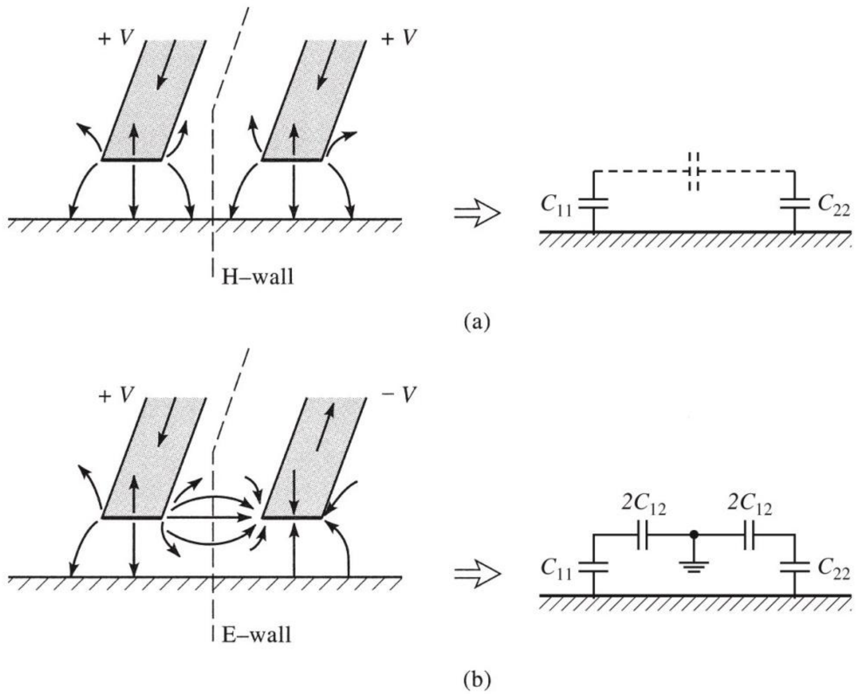
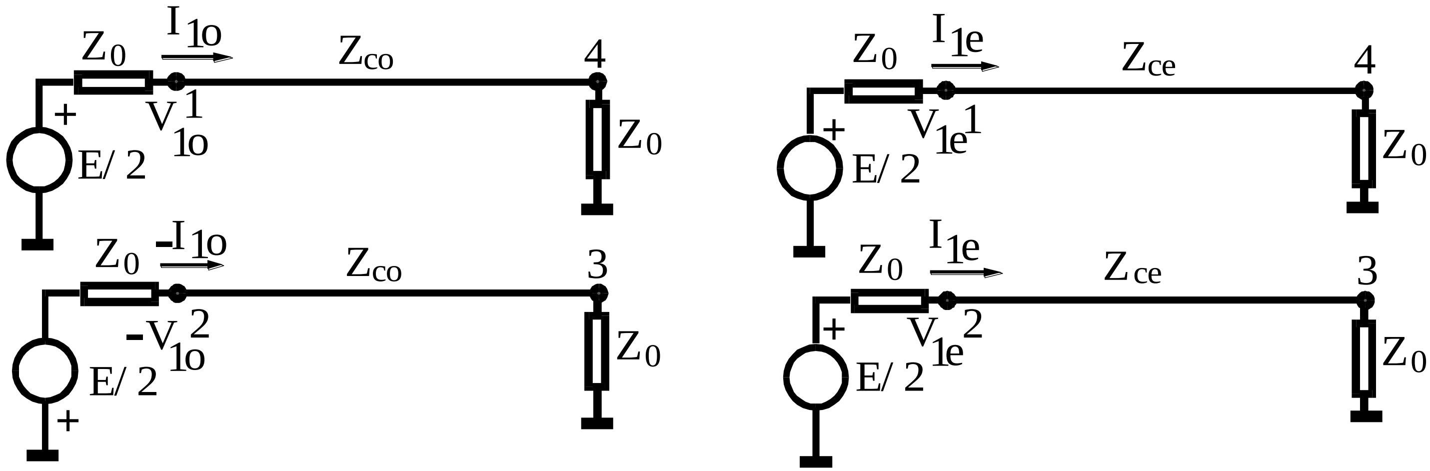


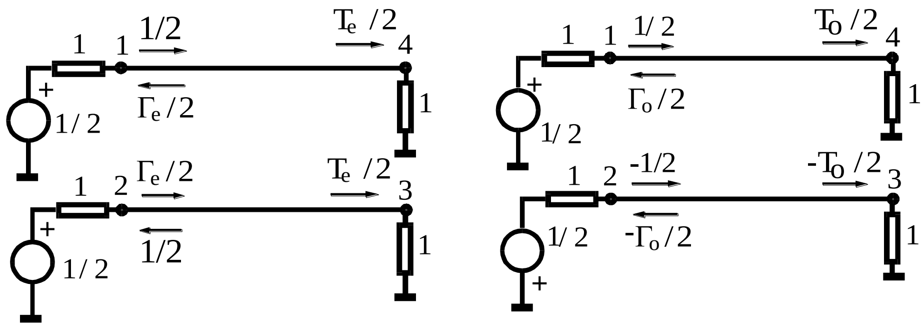
Figure 7.28

Matching in Coupled Line Coupler



$$\begin{cases} Z_{ce} Z_{co} = Z_0^2 \\ \theta_e = \theta_o \end{cases}$$

Directivity and Coupling factor



modul par

modul impar

$$a_1 = a_{1e} + a_{1o} = 1\,,\; a_2 = a_3 = a_4 = 0$$

$$b_1 = \frac{1}{2}\left(\Gamma_e + \Gamma_o\right) = 0 \Leftrightarrow$$

$$b_2 = \frac{1}{2}\left(\Gamma_e - \Gamma_o\right) = \frac{jC\sin(\theta)}{\cos(\theta)\sqrt{1-C^2} + j\sin(\theta)}$$

$$b_3 = \frac{1}{2}\left(T_e - T_o\right) = 0$$

$$b_4 = \frac{1}{2}\left(T_e + T_o\right) = \frac{\sqrt{1-C^2}}{\cos(\theta)\sqrt{1-C^2} + j\sin(\theta)}$$

$$\boxed{C = \frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}}$$

$$\boxed{\theta = \pi/2}$$

$$\boxed{[S] = \begin{bmatrix} 0 & C & 0 & -j\sqrt{1-C^2} \\ C & 0 & -j\sqrt{1-C^2} & 0 \\ 0 & -j\sqrt{1-C^2} & 0 & C \\ -j\sqrt{1-C^2} & 0 & C & 0 \end{bmatrix}}$$

Coupled Line Coupler

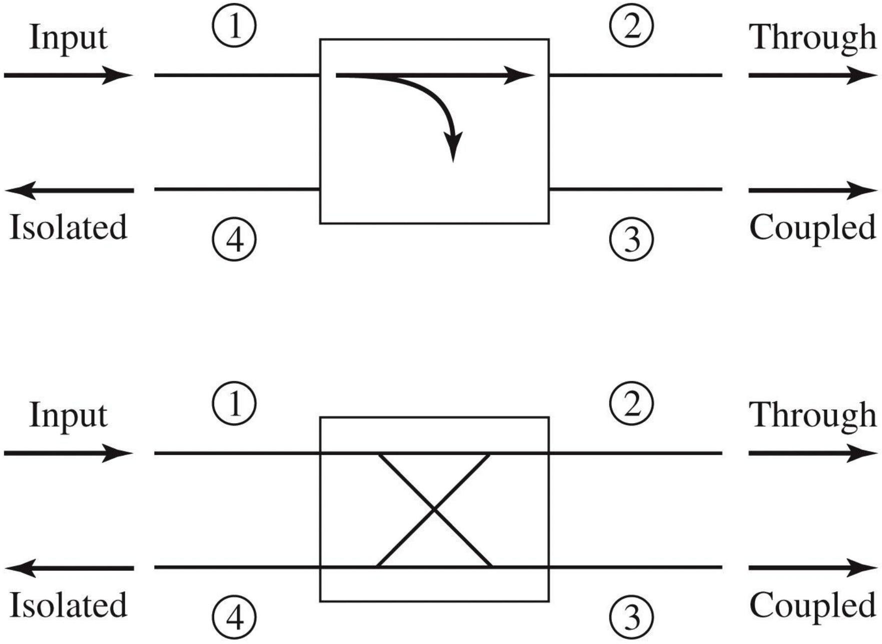
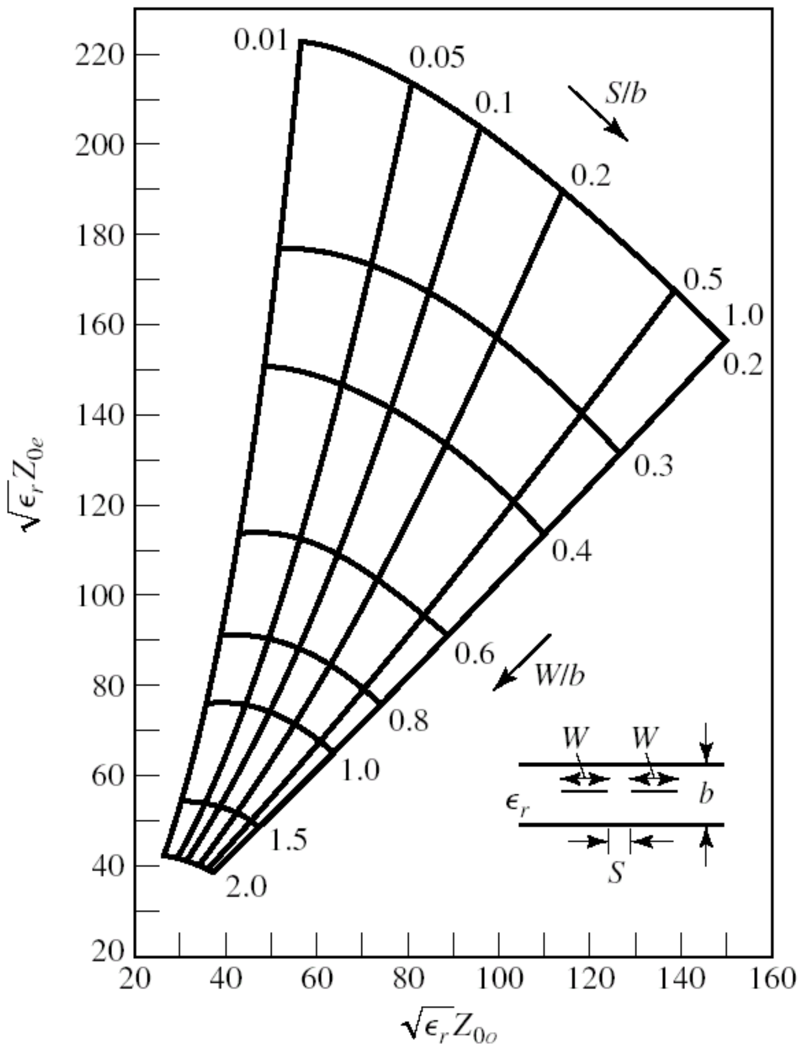


Figure 7.4

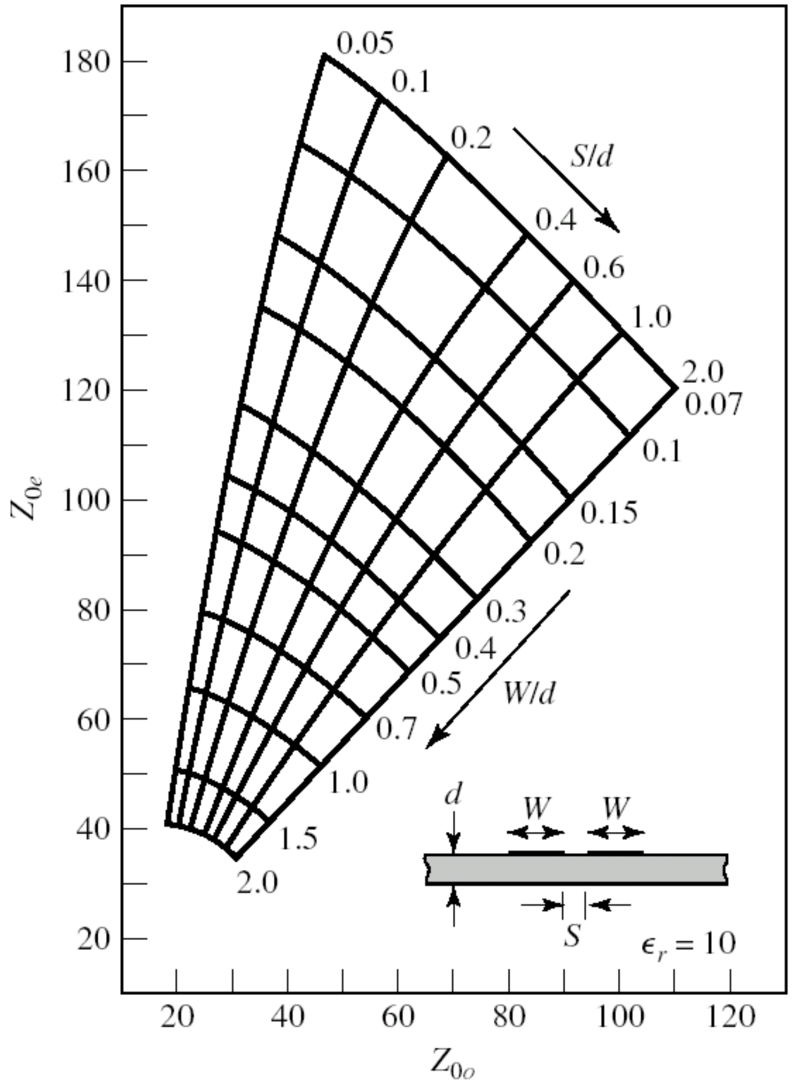
$$[S] = -j \cdot \begin{bmatrix} 0 & \sqrt{1-C^2} & jC & 0 \\ \sqrt{1-C^2} & 0 & 0 & jC \\ jC & 0 & 0 & \sqrt{1-C^2} \\ 0 & jC & \sqrt{1-C^2} & 0 \end{bmatrix}$$

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

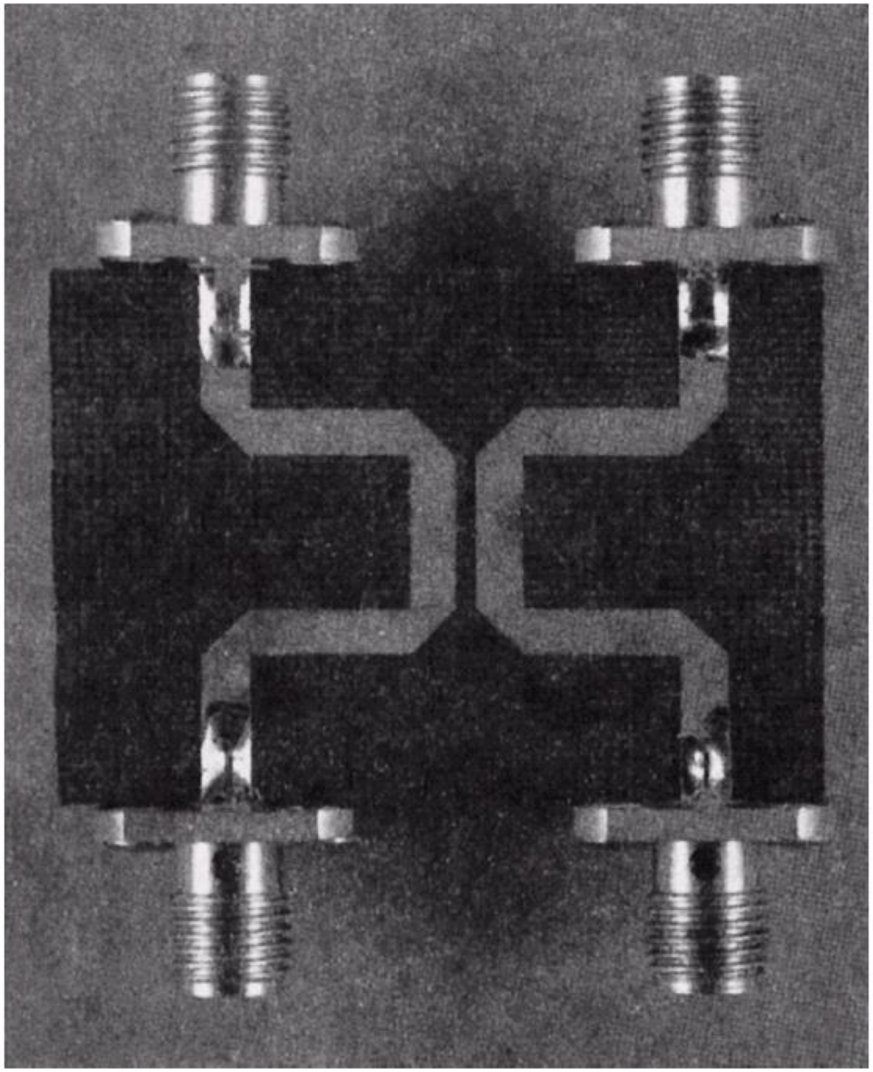
Normalized even- and odd-mode characteristic impedance design data for edge-coupled striplines.



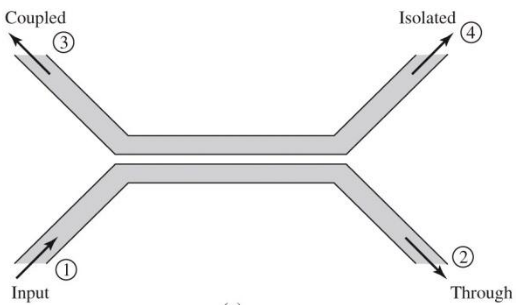
Even- and odd-mode characteristic impedance design data for coupled microstrip lines on a substrate with ε_r = 10.



Coupled Line Coupler



Coupled Line Coupler



$$\boxed{Z_{ce} Z_{co} = Z_0^2}$$

$$|\beta| = \frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}$$

$$\boxed{C\,[\mathrm{dB}] = -20 \cdot \log_{10}\left(\frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}\right)}$$

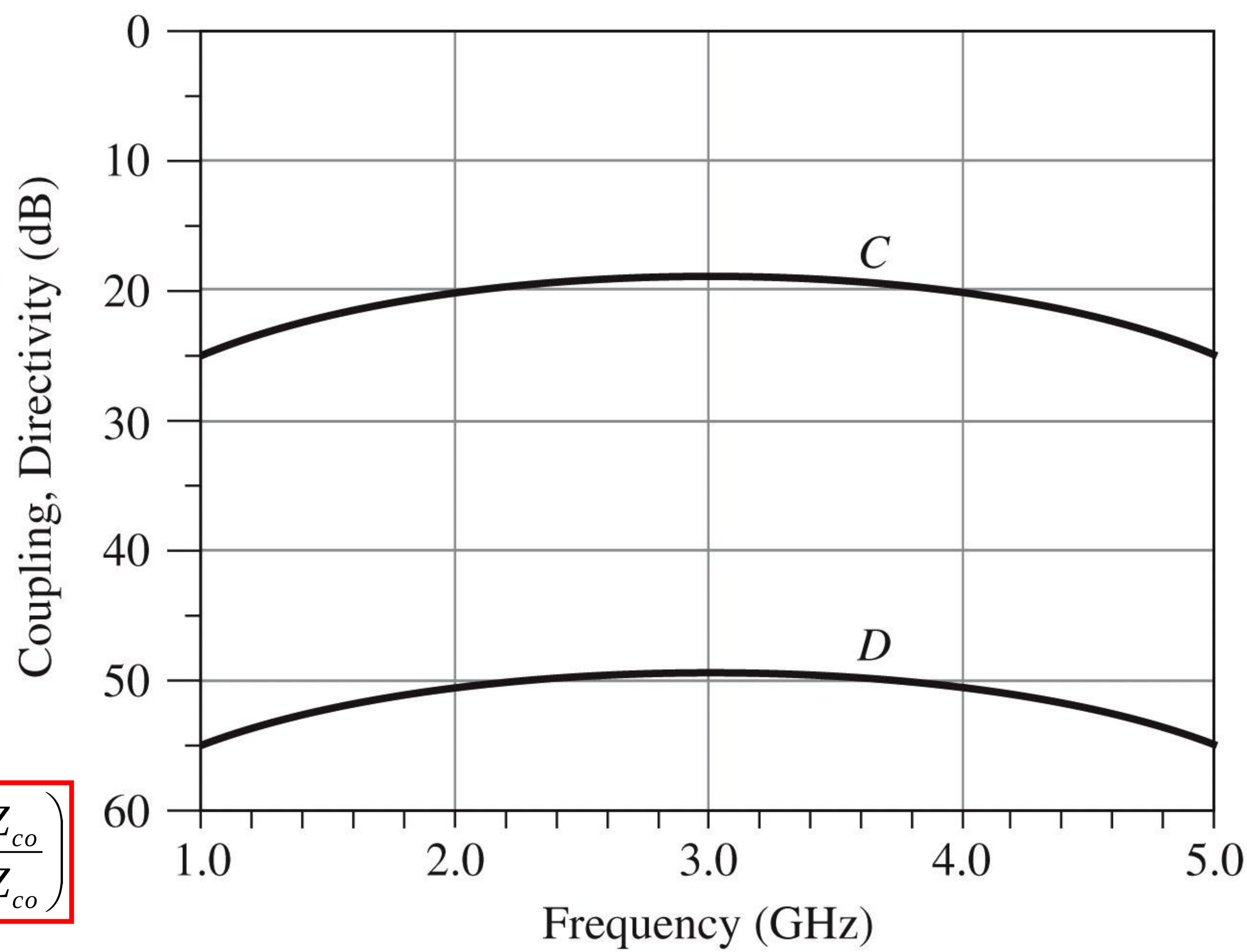


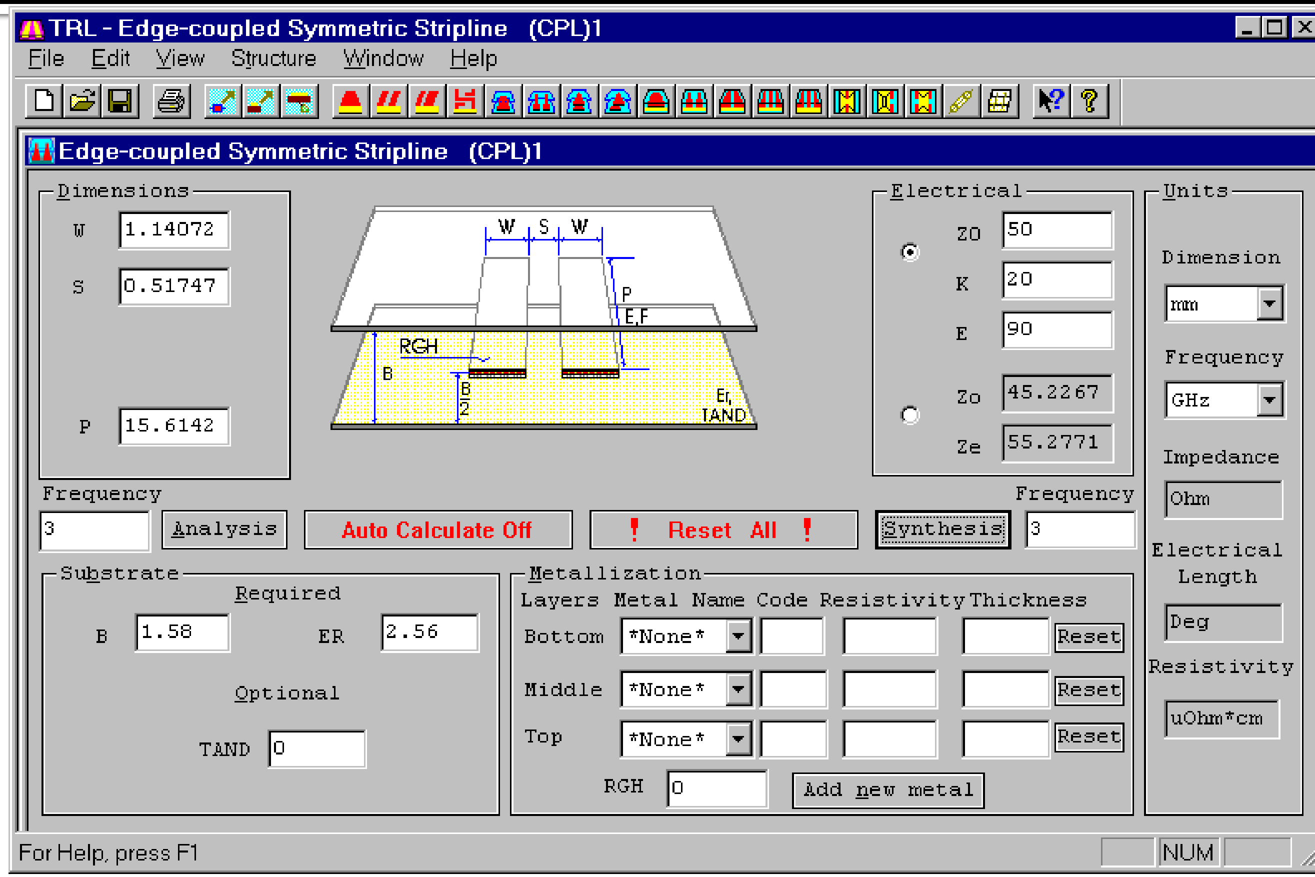
Figure 7.34

Example

Design a coupled line coupler with 20 dB coupling factor, using stripline technology, with a distance between ground planes of 0.158 cm and an electrical permittivity of 2.56, working on 50Ω, at the design frequency of 3 GHz. Plot the coupling and directivity between 1 and 5 GHz.

Solution

$$C = 10^{-20/20} = 0.1$$

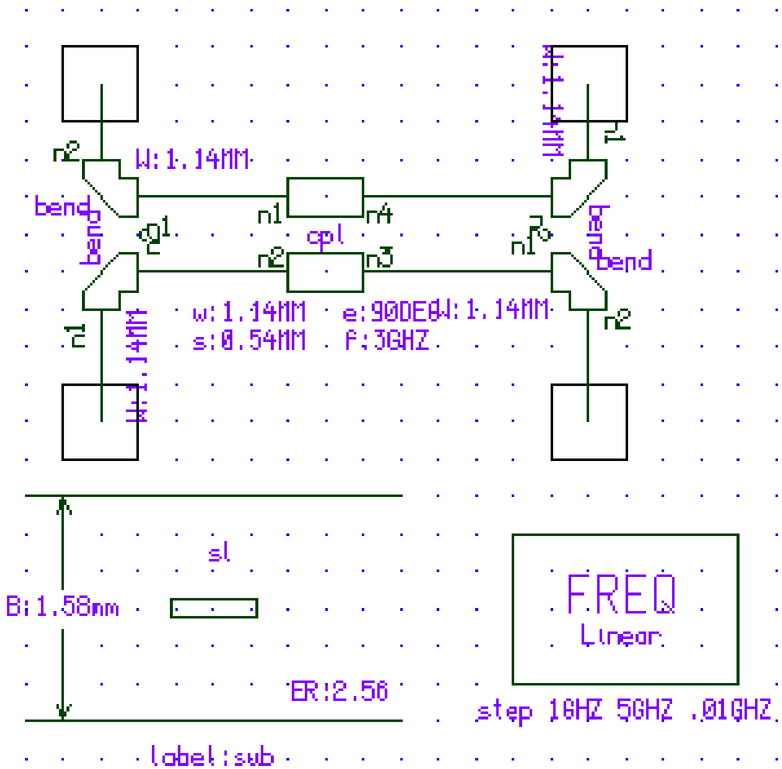


$$Z_{co} = 50\sqrt{\frac{0.9}{1.1}} = 45.23\Omega$$

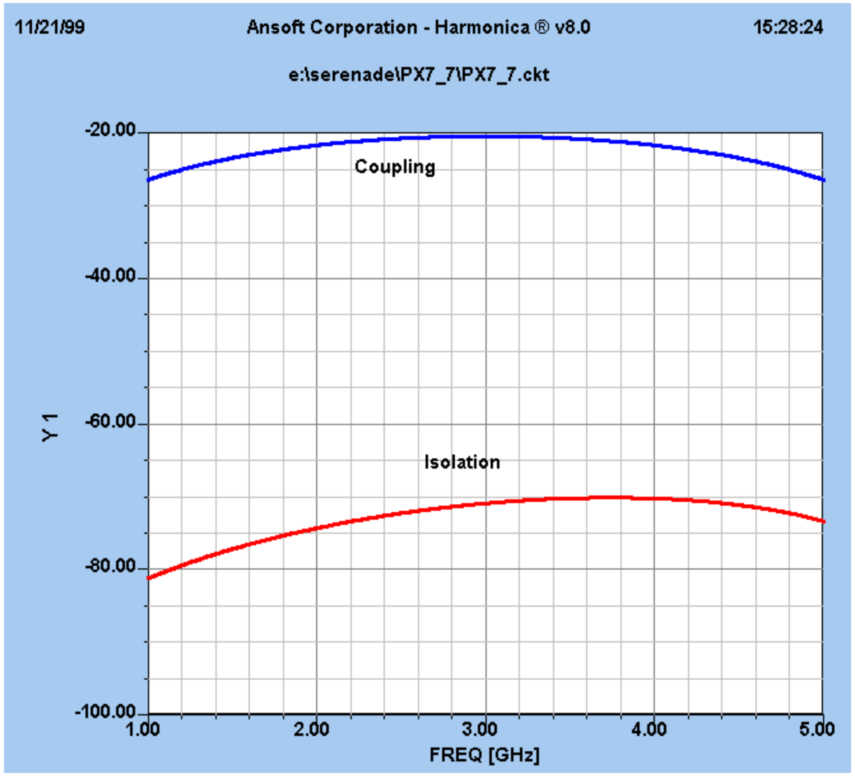
$$Z_{ce} = 50\sqrt{\frac{1.1}{0.9}} = 55.28\Omega$$

$$Z_{ce} = Z_0\sqrt{\frac{1+C}{1-C}},\ Z_{co} = Z_0\sqrt{\frac{1-C}{1+C}}$$

Simulation



W:1.14MM
bend
n1
n2
n4
n3
cpl
w:1.14MM
s:0.54MM
e:90DEG
f:3GHZ
W:1.14MM
B:1.58mm
sl
ER:2.56
label:sub
FREQ
Linear
step 1GHZ 5GHZ .01GHZ



11/21/99
Ansoft Corporation - Harmonica ® v8.0
15:28:24
e:\serenade\PX7_7\PX7_7.ckt
Coupling
Isolation
Y 1
-20.00
-40.00
-60.00
-80.00
-100.00
1.00
2.00
3.00
4.00
5.00
FREQ [GHz]

Multisection Coupled Line Couplers

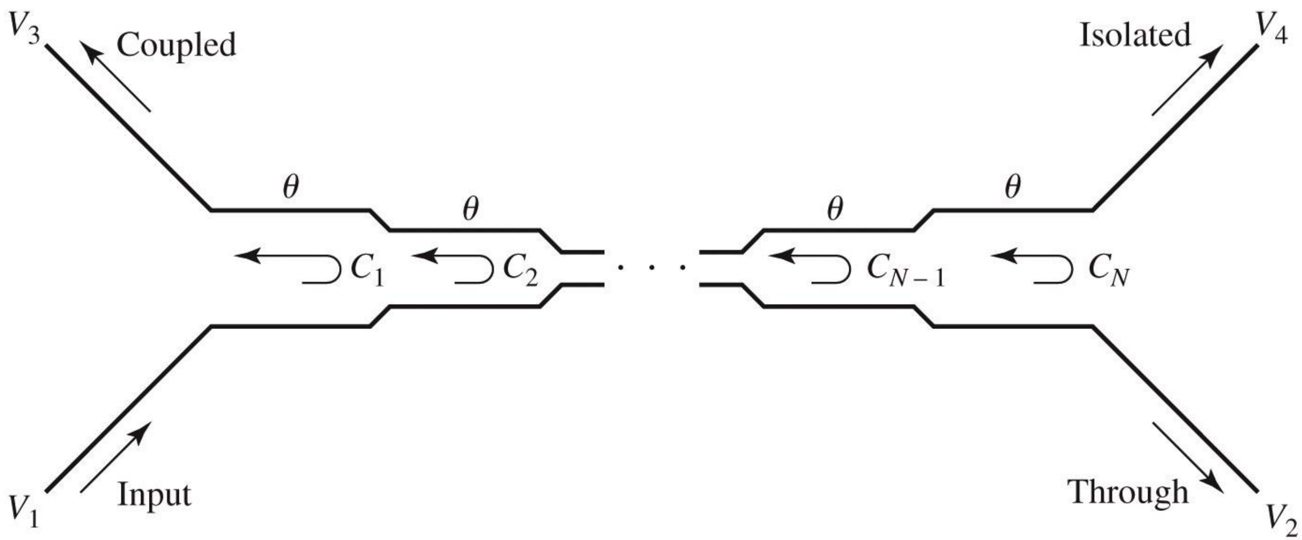


Figure 7.35

C<<1

$$\frac{V_3}{V_1} = b_3 = \frac{jC\sin\theta}{\cos\theta\sqrt{1-C^2} + j\sin\theta} = \frac{jC\mathrm{tg}\theta}{\sqrt{1-C^2} + j\mathrm{tg}\theta} \approx \frac{jC\mathrm{tg}\theta}{1 + j\mathrm{tg}\theta} = jC\sin\theta e^{-j\theta}$$

$$\frac{V_2}{V_1} = b_2 = \frac{\sqrt{1-C^2}}{\cos\theta\sqrt{1-C^2} + j\sin\theta} \approx \frac{1}{\cos\theta + j\sin} = e^{-j\theta}$$

$$C = \frac{V_3}{V_1} = 2j\sin\theta e^{-j\theta} e^{-j(N-1)\theta}\left[C_1\cos(N-1)\theta + C_2\cos(N-3)\theta + \ldots + \frac{1}{2}C_{\frac{N+1}{2}}\right]$$

Example

Design a three sections coupled line coupler with 20 dB coupling factor, binomial characteristic (maximum flat), working on 50Ω, at the design frequency of 3 GHz. Plot the coupling and directivity between 1 and 5 GHz

Solution

$$\frac{d^n}{d\theta^n} C(\theta)\Big|_{\theta=\pi/2} = 0,\ n = 1,2$$

$$C = \left|\frac{V_3}{V_1}\right| = 2\sin\theta\left[C_1\cos 2\theta + \frac{1}{2}C_2\right] = C_1(\sin 3\theta - \sin\theta) + C_2\sin\theta$$

$$\frac{dC}{d\theta} = \left[3C_1\cos 3\theta + (C_2 - C_1)\cos\theta\right]\Big|_{\theta=\pi/2} = 0$$

$$\frac{d^2C}{d\theta^2} = \left[-9C_1\sin 3\theta - (C_2 - C_1)\sin\theta\right]\Big|_{\theta=\pi/2} = 10C_1 - C_2 = 0$$

$$\begin{cases} C_2 - 2C_1 = 0.1 \\ 10C_1 - C_2 = 0 \end{cases}$$

$$\begin{cases} C_1 = C_3 = 0.0125 \\ C_2 = 0.125 \end{cases}$$

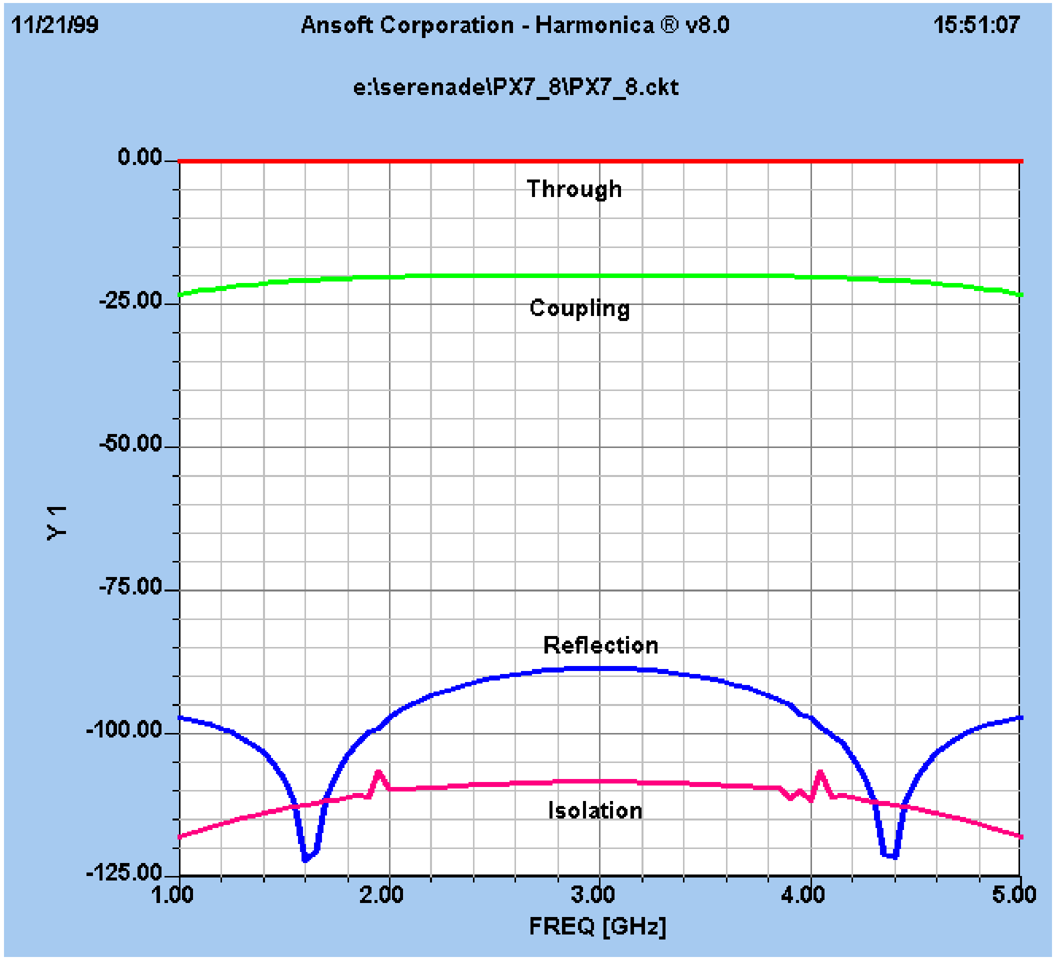
$$Z_{0e}^1 = Z_{0e}^3 = 50\sqrt{\frac{1.0125}{0.9875}} = 50.63\Omega$$

$$Z_{0o}^1 = Z_{0o}^3 = 50\sqrt{\frac{0.9875}{1.0125}} = 49.38\Omega$$

$$Z_{0e}^2 = 50\sqrt{\frac{1.125}{0.875}} = 56.69\Omega$$

$$Z_{0o}^2 = 50\sqrt{\frac{0.875}{1.125}} = 44.10\Omega$$

Simulare


11/21/99
Ansoft Corporation - Harmonica ® v8.0
15:51:07
e:\serenade\PX7_8\PX7_8.ckt
0.00
-25.00
-50.00
-75.00
-100.00
-125.00
Y 1
Through
Coupling
Reflection
Isolation
1.00
2.00
3.00
4.00
5.00
FREQ [GHz]

The Lange Coupler

- allows achieving coupling factors of 3 or 6 dB

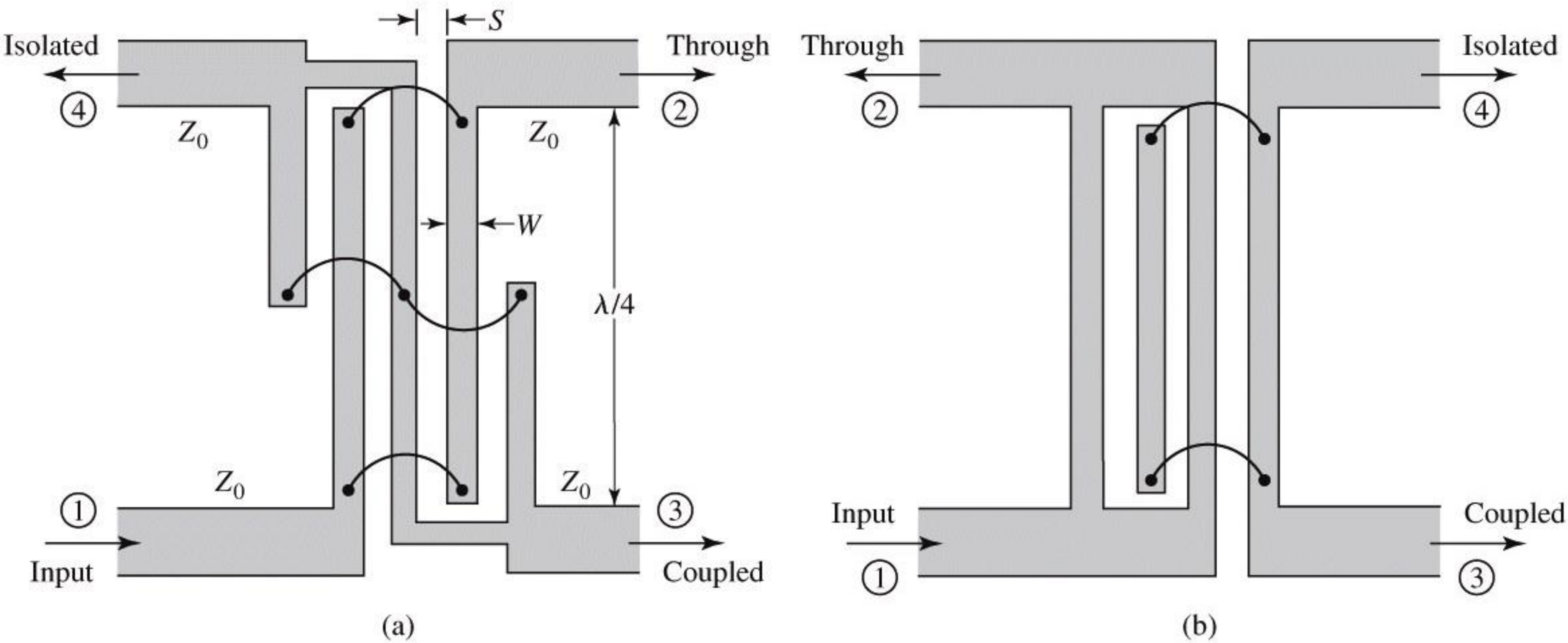


Figure 7.38

The Lange Coupler

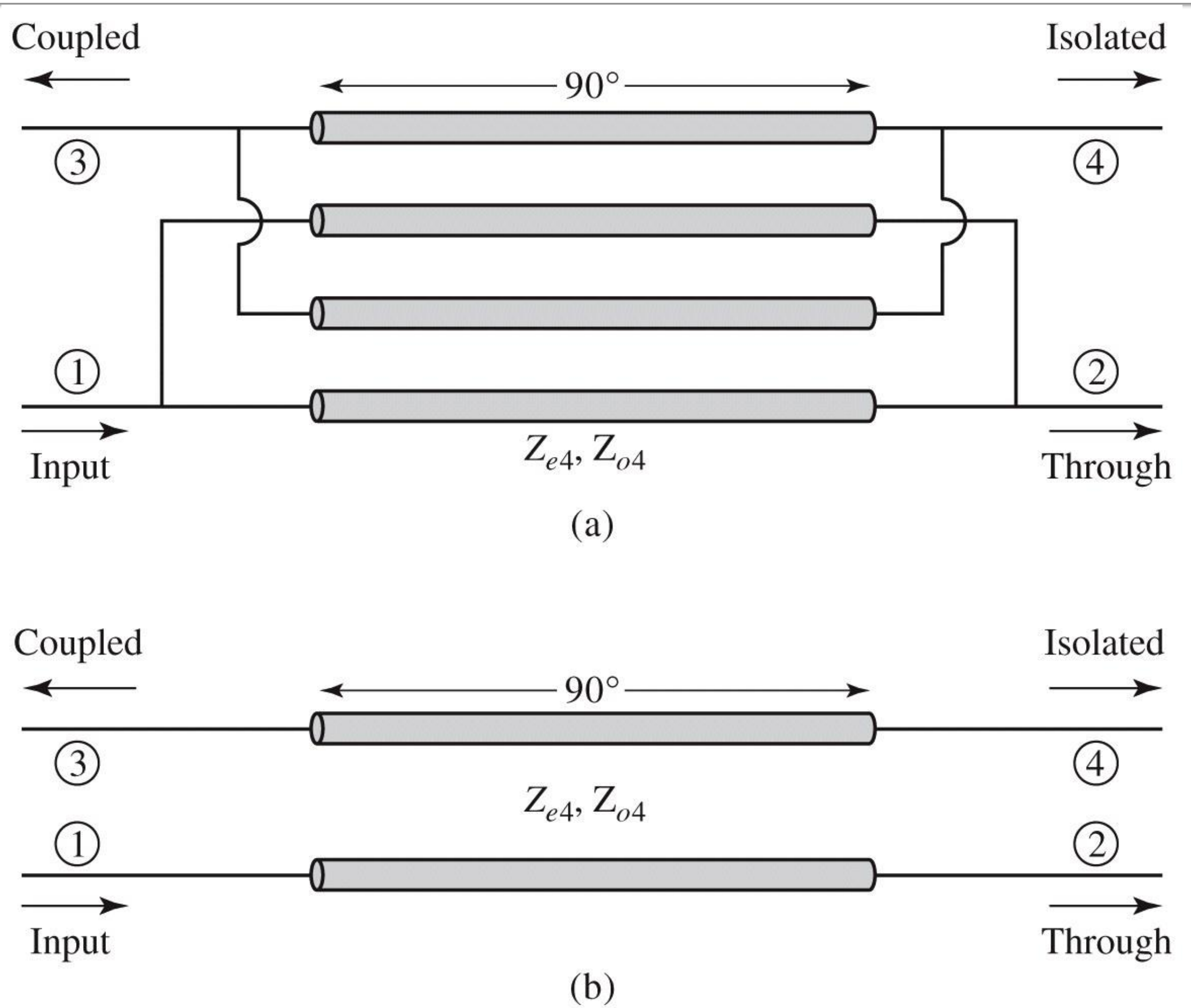
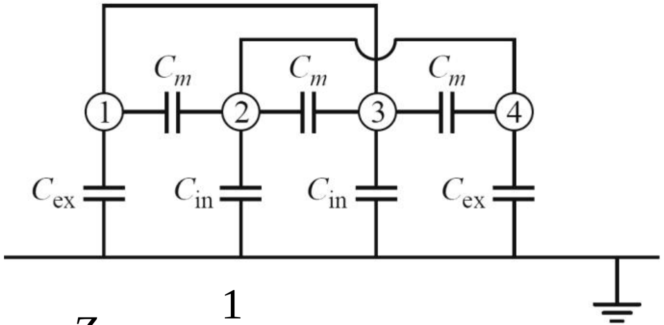


Figure 7.39

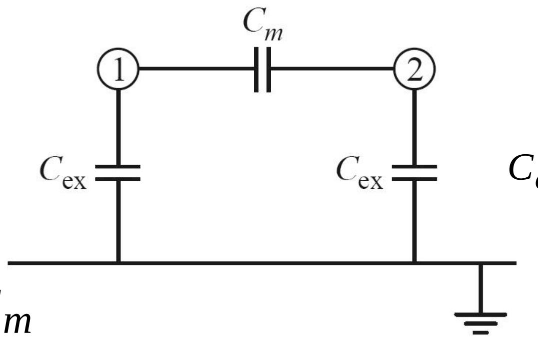
Circuit model



$$C_{in} = C_{ex} - \frac{C_{ex}C_m}{C_{ex} + C_m}$$

$$C_{e4} = C_{ex} + C_{in}$$

$$C_{o4} = C_{ex} + C_{in} + 6C_m$$



$$C_e = C_{ex}$$

$$C_o = C_{ex} + 2C_m$$

$$Z_{e4} = \frac{1}{vC_{e4}}$$

$$Z_{o4} = \frac{1}{vC_{o4}}$$

$$Z_0 = \sqrt{Z_{e4}Z_{o4}} = \sqrt{\frac{Z_{0e}Z_{0o}(Z_{0o} + Z_{0e})^2}{(3Z_{0o} + Z_{0e})(3Z_{0e} + Z_{0o})}}$$

$$C_{e4} = \frac{C_e(3C_e + C_o)}{C_e + C_o}$$

$$Z_{e4} = Z_{0e}\frac{Z_{0e} + Z_{0o}}{3Z_{0o} + Z_{0e}}$$

$$C = \frac{Z_{e4} - Z_{o4}}{Z_{e4} + Z_{o4}} = \frac{3\left(Z_{0e}^2 - Z_{0o}^2\right)}{3\left(Z_{0e}^2 + Z_{0o}^2\right) + 2Z_{0e}Z_{0o}}$$

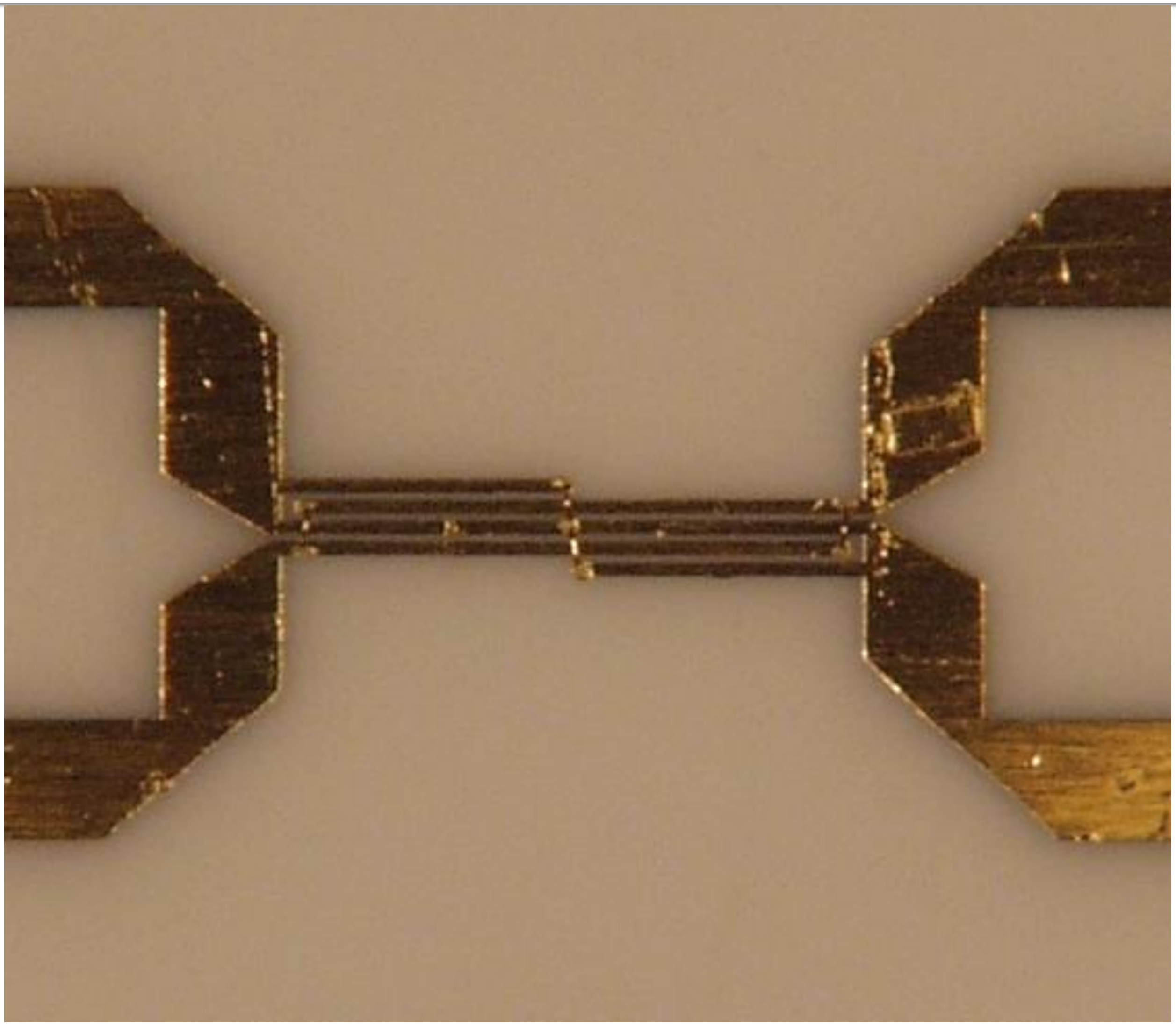
$$C_{o4} = \frac{C_o(3C_o + C_e)}{C_e + C_o}$$

$$Z_{o4} = Z_{0o}\frac{Z_{0e} + Z_{0o}}{3Z_{0e} + Z_{0o}}$$

$$\boxed{Z_{0e} = \frac{4C - 3 + \sqrt{9 - 8C^2}}{2C\sqrt{(1-C)/(1+C)}}Z_0}$$

$$\boxed{Z_{0o} = \frac{4C + 3 - \sqrt{9 - 8C^2}}{2C\sqrt{(1+C)/(1-C)}}Z_0}$$

The Lange Coupler



Directional Couplers

Laboratory no. 2

Directional Coupler

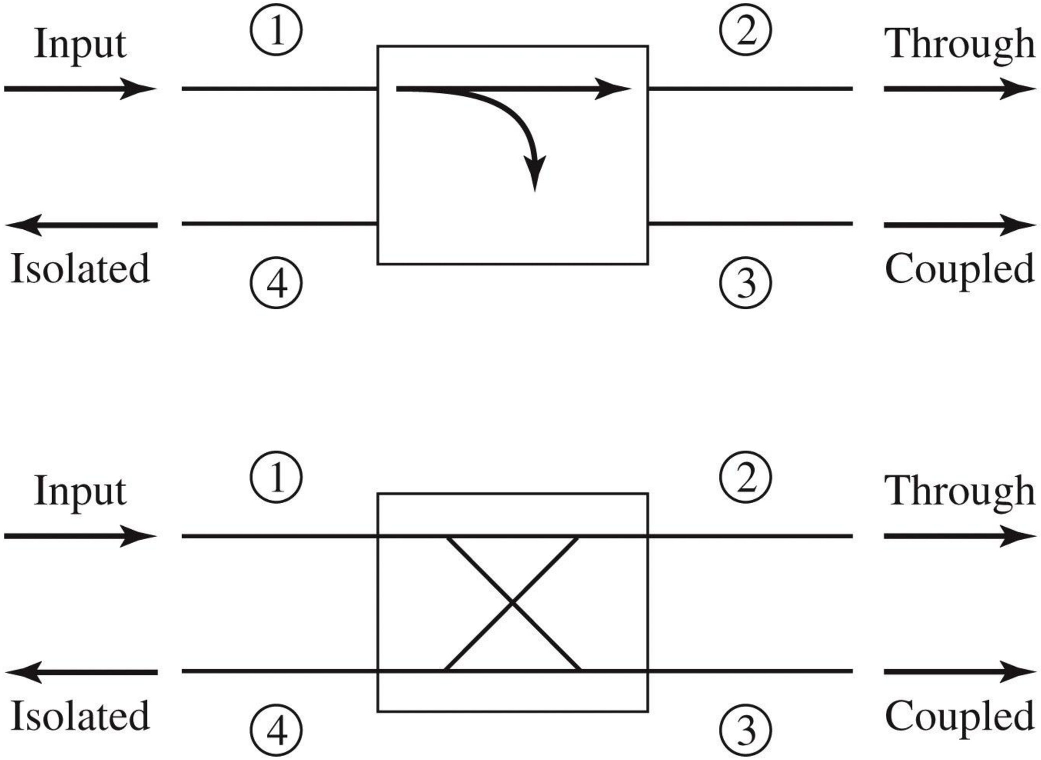


Figure 7.4

$$\left|S_{12}\right|^2 = \alpha^2 = 1 - \beta^2$$

$$\left|S_{13}\right|^2 = \beta^2$$

Cuplaj

$$C = 10\log\frac{P_1}{P_3} = -20\cdot\log(\beta)\,[\mathrm{dB}]$$

Directivitate

$$D = 10\log\frac{P_3}{P_4} = 20\cdot\log\left(\frac{\beta}{\left|S_{14}\right|}\right)[\mathrm{dB}]$$

Izolare

$$I = 10\log\frac{P_1}{P_4} = -20\cdot\log\left|S_{14}\right|\,[\mathrm{dB}]$$

$$I = D + C\;,\;\; dB$$

The cuadrature (90°) hybrid

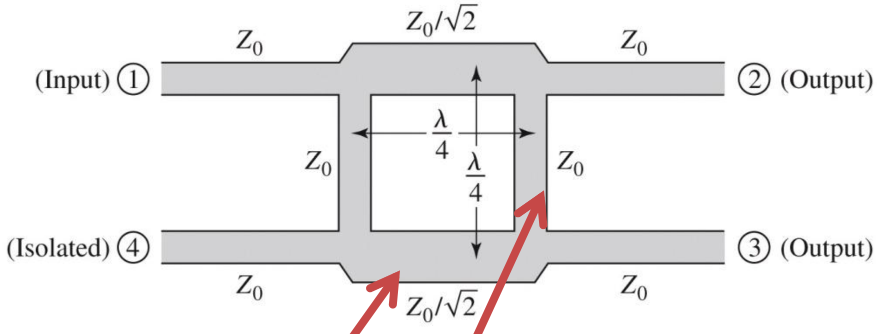


Figure 7.21

$$\boxed{y_2^2 = 1 + y_1^2}$$

$$|\beta| = \frac{\sqrt{y_2^2 - 1}}{y_2}$$

$$\boxed{C[\mathrm{dB}] = -20 \cdot \log_{10} \frac{\sqrt{y_2^2 - 1}}{y_2}}$$

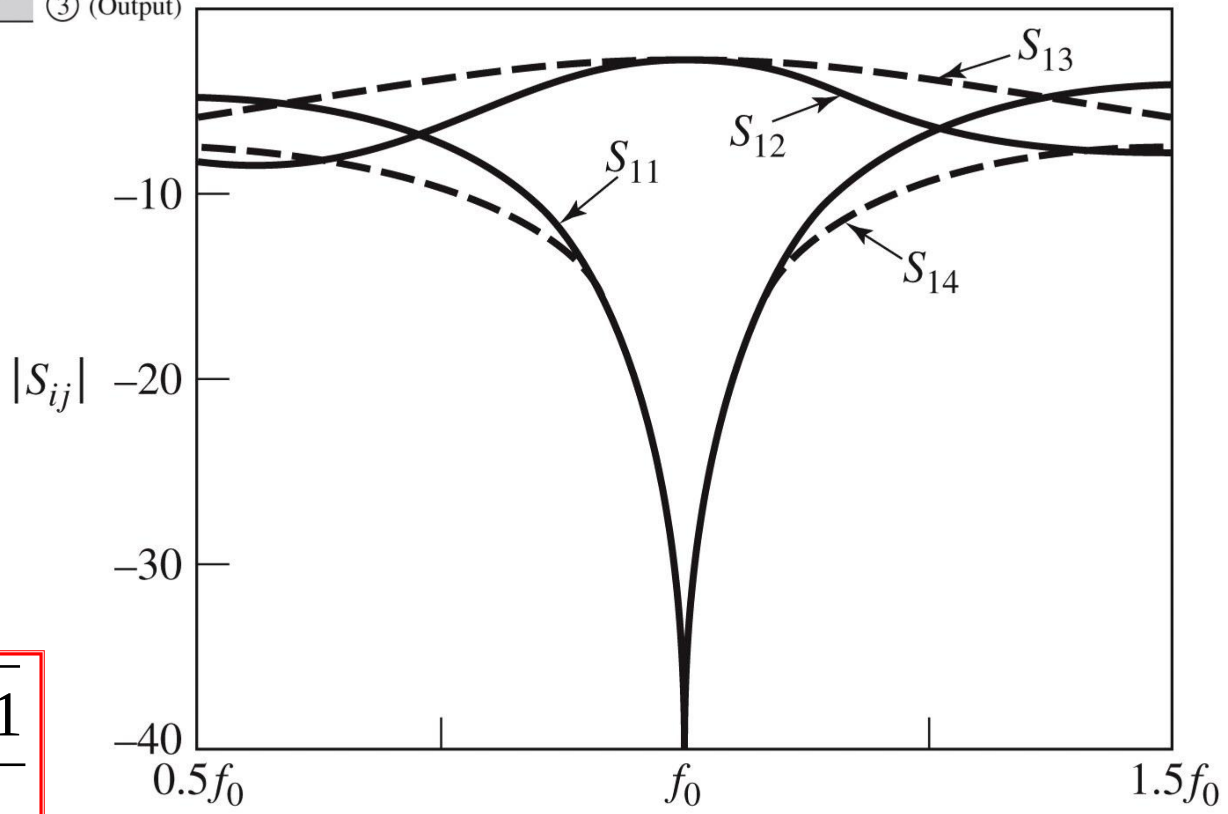
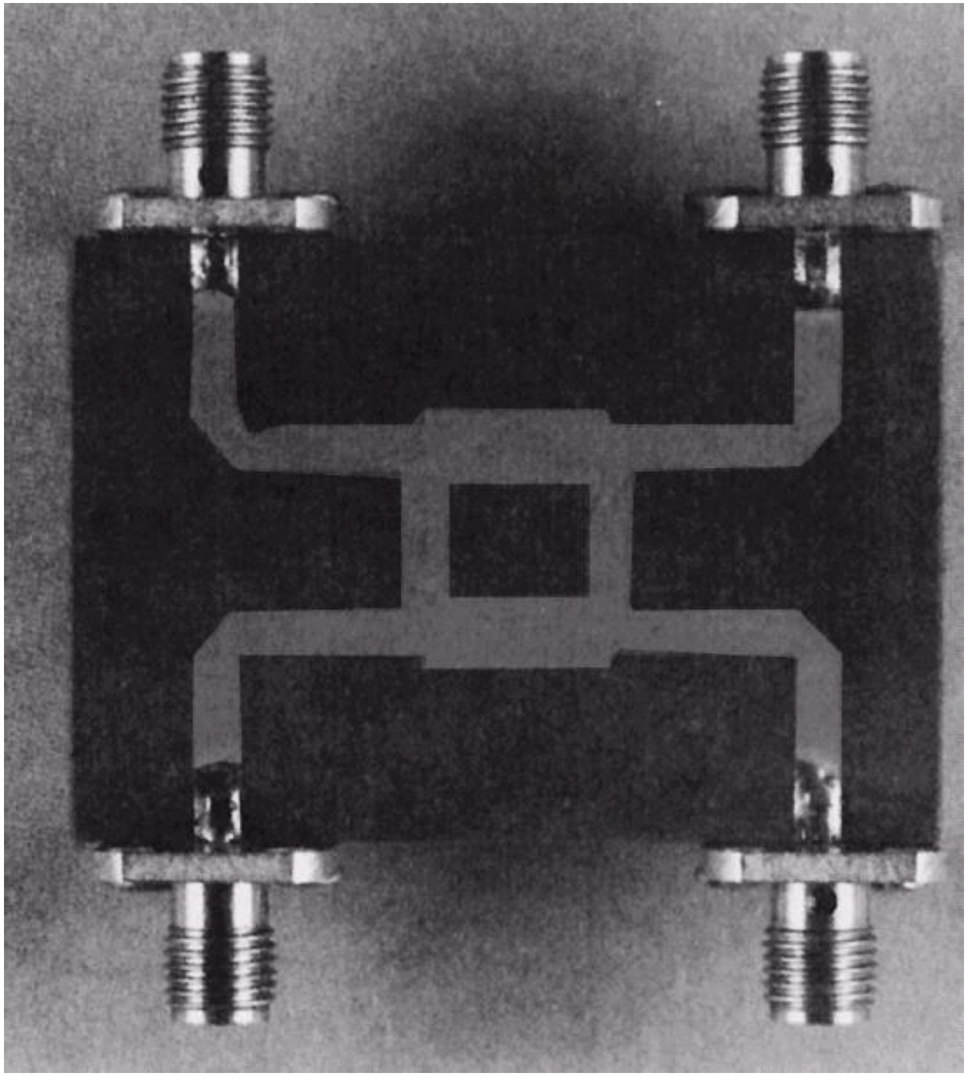
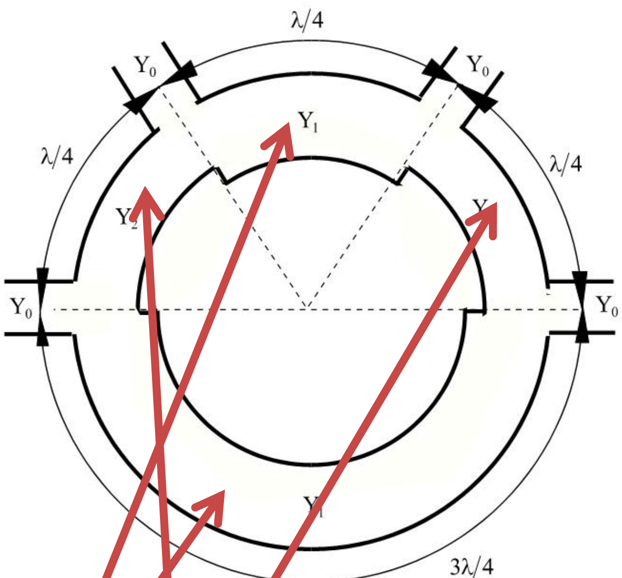


Figure 7.25

Quadrature coupler



The 180° ring hybrid (rat-race)



$$\boxed{y_1^2 + y_2^2 = 1}$$

$$|\beta| = y_1$$

$$\boxed{C\,[\mathrm{dB}] = -20 \cdot \log_{10}(y_1)}$$

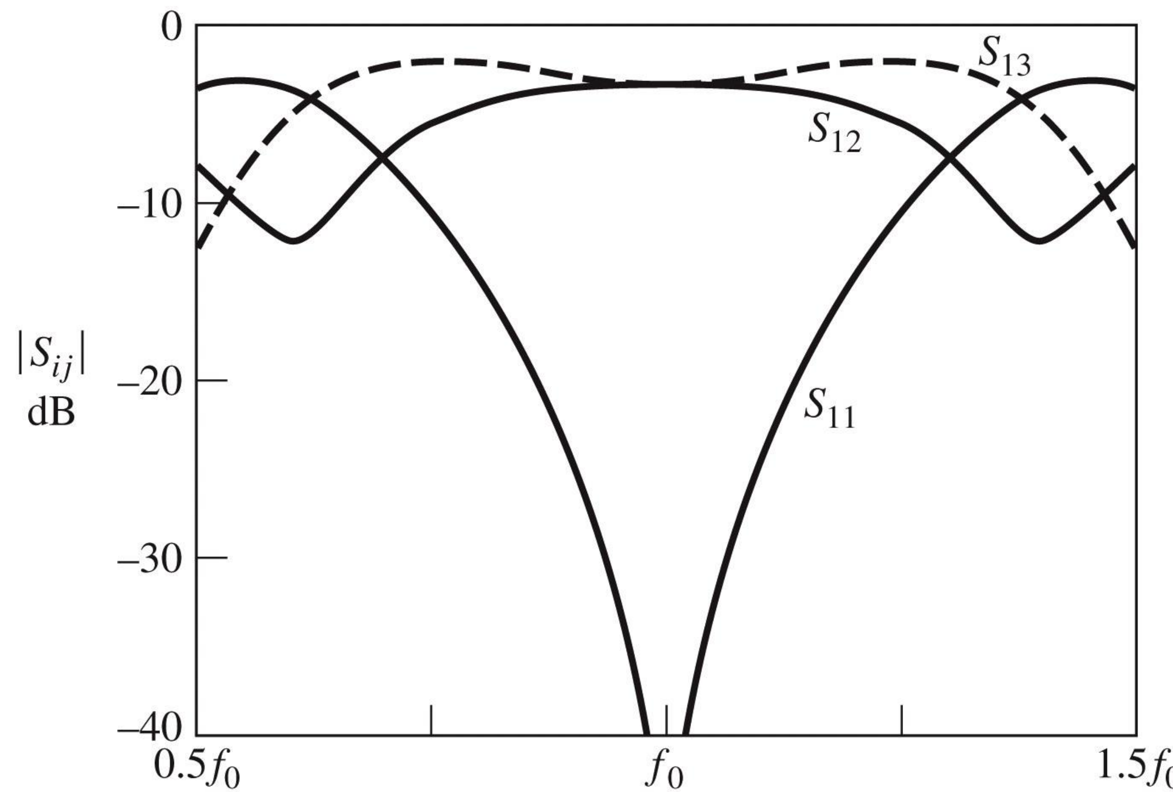


Figure 7.46

Ring coupler

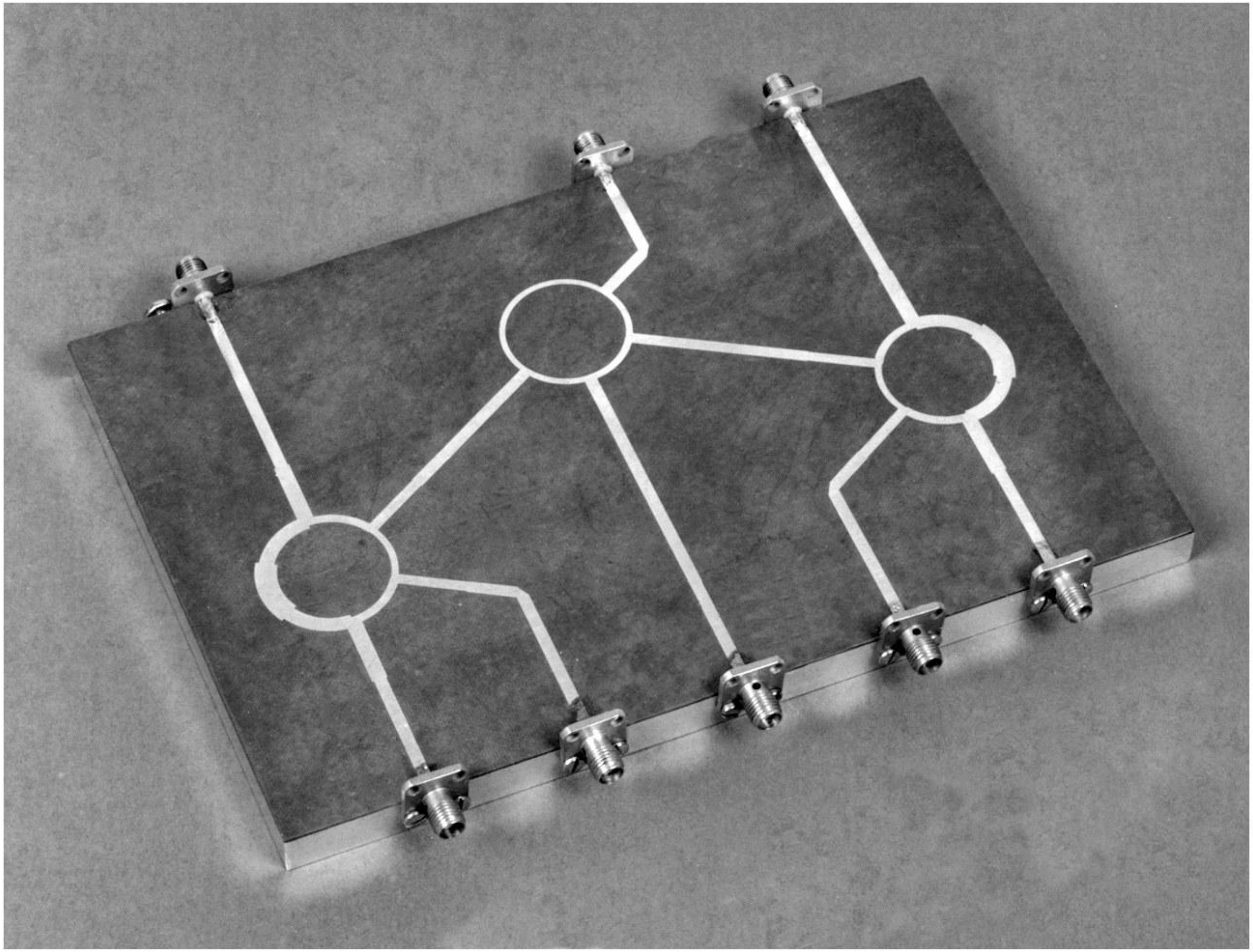
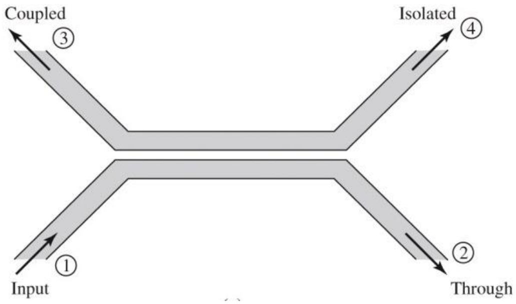


Figure 7.43
Courtesy of M. D. Abouzahra, MIT Lincoln Laboratory, Lexington, Mass.

Coupled Line Coupler



$$\boxed{Z_{ce}Z_{co} = Z_0^2}$$

$$|\beta| = \frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}$$

$$\boxed{C\,[\text{dB}] = -20 \cdot \log_{10}\left(\frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}\right)}$$

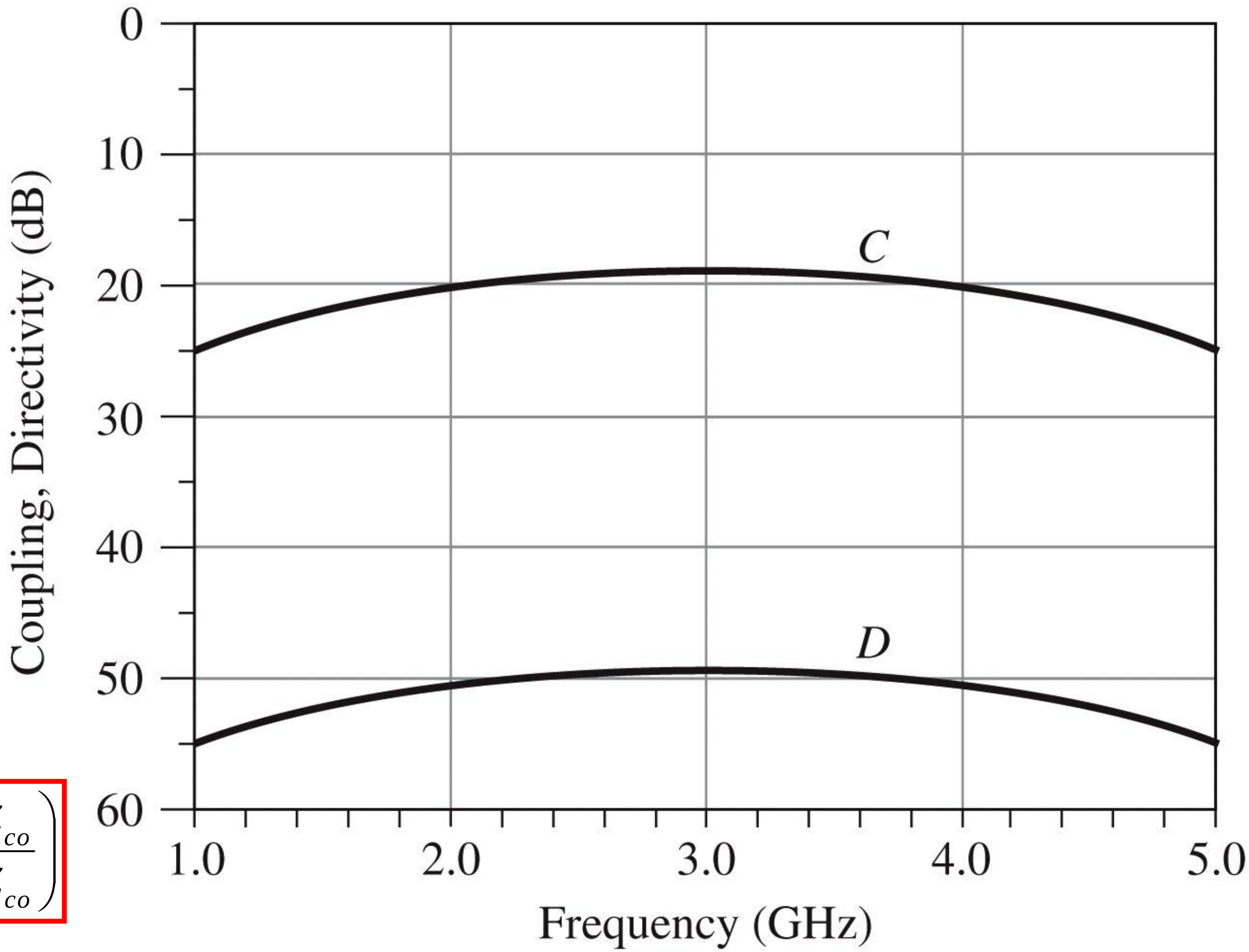
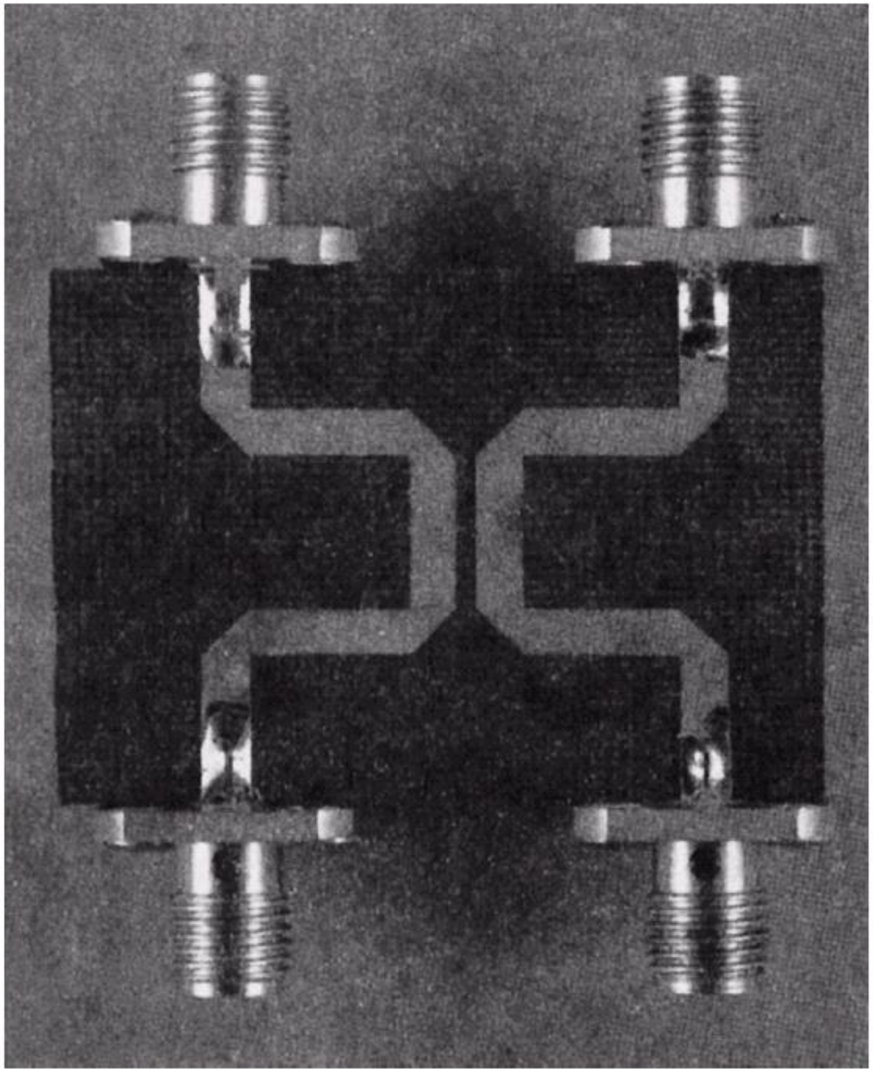


Figure 7.34

Coupled line coupler



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